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GENERALIZED FIDUCIAL INFERENCE FOR NORMAL LINEAR MIXED MODELS: SUPPLEMENTARY MATERIAL

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1. Asymptotic stability of algorithm. The stability of the algorithm was tested by considering a balanced, one-way random effects model

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}$$

where μ is a fixed effect, $\alpha_i \sim N(0, \sigma_1^2)$, $\epsilon_{ij} \sim N(0, \sigma_2^2)$, and α and ϵ are independent. Sample sizes $n = 10, 25, 50$, and 100 were considered, with $i = 1, \ldots, 5$ for $n = 10$ and 25, and $i = 1, \ldots, 10$ for $n = 50$ and 100. The number of observations within each level is such that the design is balanced (e.g. for $n = 10$, $j = 1, 2$ and for $n = 50$, $j = 1, \ldots, 5$). In addition, particle sample sizes $N = 2500, 5000$, and 10000 were considered. Three parameter designs were considered, which are listed in Table 1.

Table 1
Parameter designs for asymptotic stability test.

Parameter design	μ	σ_1^2	σ_2^2
1	0	0.75	0.25
2	0	0.25	0.75
3	0	0.50	0.50

The remaining tables in this section contain the simulation results for 95% lower, upper, and two-sided confidence intervals, and the average interval lengths for the parameter designs listed in Table 1. The study is based on 2000 independent data sets, and based on the normal approximation to the binomial distribution, empirical coverage between 94% and 96% appropriate for 95% confidence intervals.

When $n = 10$, the coverage for σ_1^2 tends to be above the stated coverage level for the lower, upper and two-sided confidence intervals except for the lower confidence intervals of parameter design 1 in which the empirical coverage is within the stated limits. Otherwise, the algorithm appears stable at all particle sample sizes considered with several cases above stated coverage and a few instances where the empirical coverage drops below the stated

coverage. The empirical coverage tends to get closer to the stated coverage as the sample size increases and the average interval lengths tend to get shorter.

TABLE 2
Coverage for 95% confidence intervals.

Parameter design	N	n	Parameter	Lower	Upper	Two	Average length
1	2500	10	μ	0.951	0.955	0.958	2.204
			σ_1^2	0.944	0.972	0.964	6.891
			σ_2^2	0.974	0.945	0.960	1.466
		25	μ	0.952	0.946	0.949	2.120
			σ_1^2	0.950	0.950	0.946	6.539
			σ_2^2	0.948	0.937	0.939	0.383
		50	μ	0.945	0.949	0.945	1.230
			σ_1^2	0.940	0.959	0.946	2.247
			σ_2^2	0.953	0.910	0.925	0.255
		100	μ	0.953	0.955	0.957	1.239
			σ_1^2	0.953	0.947	0.946	2.262
			σ_2^2	0.951	0.936	0.944	0.154
	5000	10	μ	0.953	0.957	0.954	2.192
			σ_1^2	0.946	0.974	0.966	6.812
			σ_2^2	0.976	0.938	0.960	1.442
		25	μ	0.953	0.946	0.952	2.099
			σ_1^2	0.941	0.949	0.948	6.439
			σ_2^2	0.962	0.950	0.954	0.386
		50	μ	0.948	0.938	0.947	1.240
			σ_1^2	0.945	0.960	0.958	2.273
			σ_2^2	0.957	0.929	0.938	0.248
		100	μ	0.949	0.955	0.953	1.238
			σ_1^2	0.957	0.949	0.952	2.259
			σ_2^2	0.955	0.943	0.943	0.153
	10000	10	μ	0.958	0.955	0.962	2.171
			σ_1^2	0.942	0.974	0.968	6.676
			σ_2^2	0.974	0.933	0.954	1.442
		25	μ	0.936	0.946	0.940	2.082
			σ_1^2	0.939	0.951	0.945	6.311
			σ_2^2	0.955	0.951	0.953	0.385
		50	μ	0.943	0.945	0.941	1.233
			σ_1^2	0.939	0.952	0.943	2.252
			σ_2^2	0.943	0.943	0.942	0.246
		100	μ	0.948	0.949	0.950	1.226
			σ_1^2	0.954	0.951	0.951	2.214
			σ_2^2	0.952	0.940	0.941	0.153

TABLE 3
Coverage for 95% confidence intervals.

Parameter design	N	n	Parameter	Lower	Upper	Two	Average length
2	2500	10	μ	0.977	0.980	0.986	2.041
			σ_1^2	0.999	0.986	0.993	4.712
			σ_2^2	0.954	0.968	0.968	2.630
		25	μ	0.954	0.967	0.966	1.471
			σ_1^2	0.991	0.967	0.982	2.873
			σ_2^2	0.950	0.955	0.956	1.090
		50	μ	0.950	0.951	0.944	0.864
			σ_1^2	0.944	0.958	0.956	1.074
			σ_2^2	0.956	0.941	0.946	0.726
		100	μ	0.952	0.957	0.948	0.782
			σ_1^2	0.931	0.950	0.942	0.904
			σ_2^2	0.953	0.934	0.942	0.466
	5000	10	μ	0.974	0.978	0.982	2.034
			σ_1^2	0.999	0.981	0.991	4.658
			σ_2^2	0.956	0.968	0.968	2.610
		25	μ	0.968	0.968	0.969	1.501
			σ_1^2	0.988	0.963	0.982	3.024
			σ_2^2	0.951	0.956	0.952	1.088
		50	μ	0.952	0.954	0.952	0.861
			σ_1^2	0.943	0.960	0.962	1.063
			σ_2^2	0.965	0.937	0.951	0.723
		100	μ	0.953	0.939	0.947	0.779
			σ_1^2	0.937	0.955	0.943	0.901
			σ_2^2	0.955	0.942	0.949	0.463
	10000	10	μ	0.975	0.966	0.977	2.054
			σ_1^2	0.999	0.979	0.995	4.750
			σ_2^2	0.960	0.968	0.968	2.657
		25	μ	0.967	0.958	0.973	1.497
			σ_1^2	0.986	0.964	0.983	2.999
			σ_2^2	0.956	0.951	0.955	1.100
		50	μ	0.949	0.948	0.949	0.869
			σ_1^2	0.953	0.960	0.961	1.087
			σ_2^2	0.960	0.940	0.958	0.723
		100	μ	0.956	0.953	0.952	0.787
			σ_1^2	0.933	0.946	0.936	0.918
			σ_2^2	0.958	0.934	0.950	0.464

TABLE 4
Coverage for 95% confidence intervals.

Parameter design	N	n	Parameter	Lower	Upper	Two	Average length
3	2500	10	μ	0.964	0.964	0.978	2.103
			σ_1^2	0.980	0.982	0.989	5.677
			σ_2^2	0.973	0.950	0.965	2.161
		25	μ	0.947	0.958	0.949	1.793
			σ_1^2	0.946	0.956	0.955	4.645
			σ_2^2	0.951	0.951	0.953	0.758
		50	μ	0.958	0.940	0.944	1.052
			σ_1^2	0.920	0.960	0.939	1.655
			σ_2^2	0.963	0.926	0.942	0.506
		100	μ	0.950	0.946	0.957	1.031
			σ_1^2	0.938	0.947	0.939	1.580
			σ_2^2	0.955	0.934	0.949	0.311
	5000	10	μ	0.961	0.961	0.965	2.077
			σ_1^2	0.986	0.985	0.993	5.518
			σ_2^2	0.966	0.953	0.964	2.106
		25	μ	0.945	0.949	0.947	1.783
			σ_1^2	0.943	0.958	0.958	4.589
			σ_2^2	0.956	0.949	0.954	0.759
		50	μ	0.954	0.952	0.953	1.059
			σ_1^2	0.941	0.961	0.950	1.664
			σ_2^2	0.963	0.927	0.943	0.503
		100	μ	0.940	0.951	0.943	1.023
			σ_1^2	0.943	0.949	0.945	1.552
			σ_2^2	0.960	0.941	0.950	0.310
	10000	10	μ	0.966	0.963	0.971	2.094
			σ_1^2	0.980	0.983	0.988	5.624
			σ_2^2	0.966	0.960	0.972	2.128
		25	μ	0.950	0.950	0.949	1.786
			σ_1^2	0.943	0.955	0.953	4.611
			σ_2^2	0.949	0.946	0.948	0.759
		50	μ	0.942	0.955	0.942	1.066
			σ_1^2	0.939	0.960	0.944	1.687
			σ_2^2	0.953	0.944	0.949	0.494
		100	μ	0.953	0.950	0.950	1.029
			σ_1^2	0.944	0.948	0.944	1.572
			σ_2^2	0.951	0.940	0.945	0.307

2. Two-fold nested model. Box plot summaries for each non-error variance component for the two-fold nested model of Section 3.1 are displayed in this section. In addition, the complete results for the simulation are included. Intervals were not determined for **USS** or **TYPEI** for μ or σ_ϵ^2 because the corresponding paper did not propose a methodology to compute them. We also note that not all 2000 data sets converged for **HLMM**; the empirical coverage and average lengths for **HLMM** were computed only using the converging data sets. The percentage of converging data sets is displayed in Table 5.

TABLE 5
Two-fold nested: percentage of converged data sets for **HLMM** *(out of 2000 data sets).*

	Parameter design				
Model Design	1	2	3	4	5
1	95.80	99.15	99.80	99.05	99.20
2	95.25	98.75	99.95	98.30	98.80
3	95.40	99.55	100.00	99.05	99.55
4	94.85	99.35	99.95	99.20	98.90
5	94.75	98.40	99.15	98.35	98.10

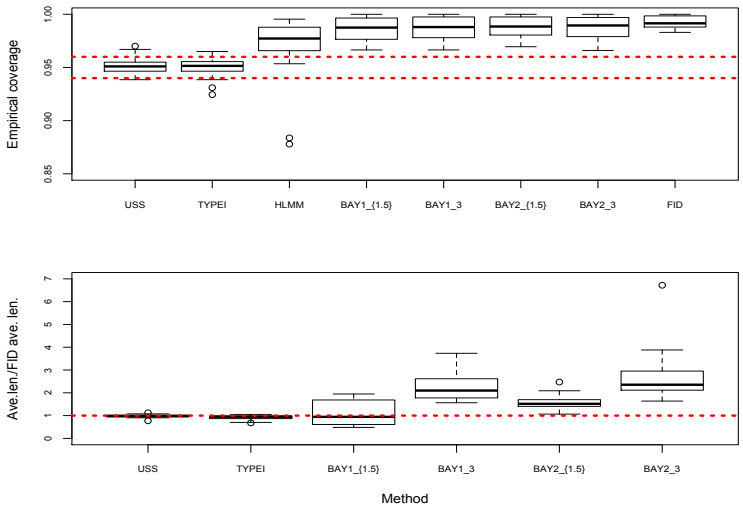

FIG 1. *Simulation results for 95% two-sided confidence intervals on σ_α^2 the two-fold nested model of Equation (3.1). The top plot is of the empirical coverage probabilities of the intervals, and the bottom plot is of the average interval lengths divided by the average interval lengths of* **FID**. *Average interval lengths are not included for* **HLMM** *because of their excessive lengths would hinder the scale of the plot.*

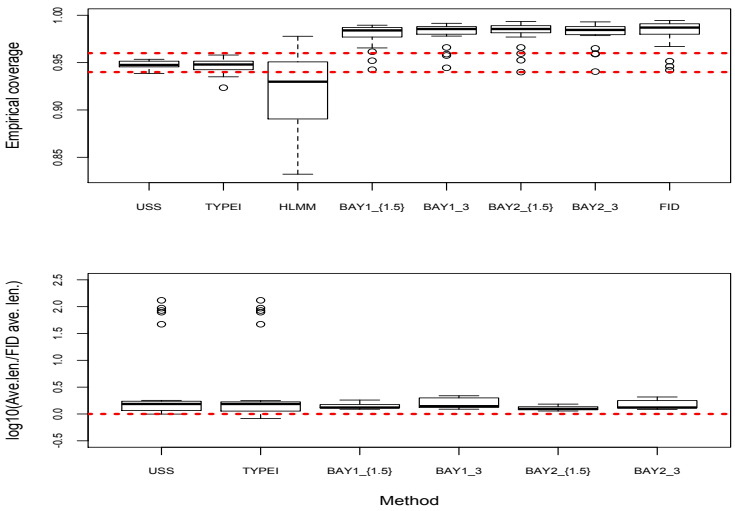

FIG 2. *Simulation results for 95% two-sided confidence intervals on σ_β^2 the two-fold nested model of Equation (3.1). The top plot is of the empirical coverage probabilities of the intervals, and the bottom plot is of the base 10 logarithm of average interval lengths divided by the average interval lengths of* **FID**. *Average interval lengths are not included for* **HLMM** *because of their excessive lengths would hinder the scale of the plot.*

TABLE 6
Two-fold nested: model design 1.

Par. design	Type	Par.	**USS**	**TYPEI**	**HLMM**	**BAY1**$_{1.5}$	**BAY1**$_{3}$	**BAY2**$_{1.5}$	**BAY2**$_{3}$	**FID**
1	Lower	μ	NA	NA	0.916	0.997	0.998	0.995	0.997	0.987
		σ^2_α	0.954	0.947	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.856	0.964	0.964	0.963	0.963	0.943
		σ^2_β	0.948	0.948	1.000	1.000	1.000	1.000	1.000	1.000
	Upper	μ	NA	NA	0.920	0.994	0.996	0.995	0.996	0.990
		σ^2_α	0.980	0.977	0.927	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.977	0.947	0.948	0.948	0.947	0.966
		σ^2_β	0.952	0.952	0.914	0.978	0.977	0.979	0.979	0.985
	Two	μ	NA	NA	0.883	0.997	0.999	0.998	0.999	0.995
		σ^2_α	0.967	0.963	0.958	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.903	0.955	0.957	0.957	0.956	0.961
		σ^2_β	0.950	0.950	0.941	0.989	0.992	0.993	0.993	0.995
	Length	μ	NA	NA	2.092E+04	2.901	3.082	2.797	2.956	2.330
		σ^2_α	4.500	4.100	∞	9.662	13.125	9.387	11.432	4.961
		σ^2_ξ	NA	NA	1.095	1.726	1.729	1.723	1.727	1.572
		σ^2_β	278.500	278.500	3.827E+111	6.417	7.397	5.391	6.483	3.531
2	Lower	μ	NA	NA	0.912	0.986	0.989	0.982	0.988	0.974
		σ^2_α	0.949	0.946	0.989	1.000	1.000	1.000	1.000	0.996
		σ^2_ξ	NA	NA	0.900	0.977	0.977	0.976	0.976	0.966
		σ^2_β	0.945	0.945	0.980	1.000	1.000	1.000	1.000	0.997
	Upper	μ	NA	NA	0.914	0.990	0.991	0.988	0.990	0.980
		σ^2_α	0.974	0.975	0.957	0.998	1.000	0.999	1.000	1.000
		σ^2_ξ	NA	NA	0.968	0.929	0.928	0.927	0.927	0.957
		σ^2_β	0.952	0.952	0.964	0.974	0.972	0.973	0.973	0.985
	Two	μ	NA	NA	0.889	0.990	0.990	0.989	0.991	0.980
		σ^2_α	0.964	0.965	0.981	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.934	0.953	0.952	0.953	0.953	0.959
		σ^2_β	0.946	0.946	0.978	0.985	0.986	0.987	0.986	0.991
	Length	μ	NA	NA	7.659E+09	3.114	3.451	3.016	3.292	2.481
		σ^2_α	6.180	5.850	∞	10.744	16.473	10.742	14.346	6.325
		σ^2_ξ	NA	NA	0.544	0.881	0.883	0.883	0.883	0.815
		σ^2_β	373.865	373.865	1.515E+27	7.375	9.615	6.450	8.107	4.408
3	Lower	μ	NA	NA	0.919	0.988	0.991	0.980	0.987	0.974
		σ^2_α	0.952	0.946	0.998	1.000	1.000	1.000	1.000	0.999
		σ^2_ξ	NA	NA	0.912	0.973	0.974	0.974	0.973	0.961
		σ^2_β	0.947	0.947	0.739	0.992	0.992	0.985	0.990	0.955
	Upper	μ	NA	NA	0.913	0.980	0.988	0.980	0.987	0.976
		σ^2_α	0.958	0.960	0.815	0.988	0.995	0.989	0.997	0.996
		σ^2_ξ	NA	NA	0.972	0.922	0.922	0.923	0.923	0.953
		σ^2_β	0.956	0.956	0.988	0.977	0.978	0.977	0.979	0.989
	Two	μ	NA	NA	0.892	0.986	0.994	0.984	0.993	0.982
		σ^2_α	0.951	0.952	0.884	0.997	1.000	0.997	1.000	1.000
		σ^2_ξ	NA	NA	0.933	0.941	0.939	0.937	0.939	0.959
		σ^2_β	0.953	0.953	0.832	0.985	0.986	0.984	0.985	0.980
	Length	μ	NA	NA	7.075E+10	3.095	3.763	3.006	3.509	2.626
		σ^2_α	6.346	5.945	∞	10.408	18.231	10.286	15.243	6.851
		σ^2_ξ	NA	NA	0.193	0.316	0.316	0.317	0.316	0.296
		σ^2_β	716.537	716.537	8.423E+53	8.166	11.932	7.608	9.878	5.463
4	Lower	μ	NA	NA	0.918	0.983	0.990	0.983	0.988	0.973
		σ^2_α	0.948	0.939	0.964	0.998	1.000	0.998	0.999	0.981
		σ^2_ξ	NA	NA	0.875	0.963	0.964	0.962	0.962	0.949
		σ^2_β	0.945	0.945	1.000	1.000	1.000	1.000	1.000	1.000
	Upper	μ	NA	NA	0.909	0.982	0.985	0.979	0.983	0.969
		σ^2_α	0.987	0.985	0.985	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.976	0.937	0.937	0.937	0.938	0.957
		σ^2_β	0.952	0.952	0.884	0.965	0.970	0.972	0.968	0.982
	Two	μ	NA	NA	0.878	0.990	0.992	0.988	0.992	0.979
		σ^2_α	0.970	0.963	0.986	1.000	1.000	1.000	1.000	0.996
		σ^2_ξ	NA	NA	0.897	0.948	0.946	0.945	0.949	0.952
		σ^2_β	0.951	0.951	0.930	0.987	0.989	0.989	0.988	0.992
	Length	μ	NA	NA	2.088E+10	19.862	21.643	19.240	20.710	15.993
		σ^2_α	277.313	266.274	∞	461.588	677.322	441.821	579.218	273.864
		σ^2_ξ	NA	NA	27.465	43.924	43.914	41.413	41.391	40.479
		σ^2_β	8054.500	8054.500	8.899E+66	292.438	354.273	254.717	308.478	171.342
5	Lower	μ	NA	NA	0.916	0.986	0.989	0.984	0.989	0.979
		σ^2_α	0.951	0.947	0.995	1.000	1.000	1.000	1.000	0.995
		σ^2_ξ	NA	NA	0.876	0.972	0.972	0.970	0.971	0.956
		σ^2_β	0.952	0.952	0.976	1.000	1.000	1.000	1.000	0.995
	Upper	μ	NA	NA	0.925	0.981	0.989	0.986	0.987	0.975
		σ^2_α	0.970	0.972	0.934	0.999	0.999	0.999	0.999	1.000
		σ^2_ξ	NA	NA	0.973	0.932	0.933	0.933	0.933	0.956
		σ^2_β	0.949	0.949	0.971	0.972	0.974	0.976	0.975	0.984
	Two	μ	NA	NA	0.899	0.992	0.995	0.990	0.992	0.981
		σ^2_α	0.960	0.960	0.971	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.915	0.951	0.952	0.951	0.952	0.956
		σ^2_β	0.953	0.953	0.977	0.988	0.988	0.990	0.989	0.991
	Length	μ	NA	NA	7.197E+08	5.429	5.997	5.271	5.749	4.340
		σ^2_α	18.337	17.109	∞	32.945	49.191	31.395	43.061	18.807
		σ^2_ξ	NA	NA	1.771	2.878	2.879	2.879	2.876	2.658
		σ^2_β	1297.000	1297.000	6.823E+105	22.918	28.877	20.316	25.544	13.807

Simulation results for 95% confidence intervals. Intervals were not determined for **USS** or **TYPEI** for μ or σ^2_ϵ.

TABLE 7
Two-fold nested: model design 2.

Par. design	Type	Par.	**USS**	**TYPEI**	**HLMM**	**BAY1**$_{1.5}$	**BAY1**$_3$	**BAY2**$_{1.5}$	**BAY2**$_3$	**FID**
1	Lower	μ	NA	NA	0.897	1.000	0.999	0.999	1.000	0.995
		σ^2_α	0.946	0.940	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.890	0.965	0.964	0.964	0.962	0.948
		σ^2_β	0.938	0.946	1.000	1.000	1.000	1.000	1.000	1.000
	Upper	μ	NA	NA	0.891	1.000	1.000	0.998	1.000	0.998
		σ^2_α	0.952	0.966	0.945	0.971	0.964	0.972	0.968	0.995
		σ^2_ξ	NA	NA	0.981	0.953	0.953	0.954	0.954	0.965
		σ^2_β	0.948	0.955	0.908	0.961	0.965	0.965	0.966	0.976
	Two	μ	NA	NA	0.852	1.000	1.000	1.000	1.000	0.999
		σ^2_α	0.952	0.953	0.974	0.993	0.992	0.992	0.990	0.999
		σ^2_ξ	NA	NA	0.923	0.958	0.957	0.957	0.958	0.952
		σ^2_β	0.939	0.951	0.941	0.983	0.983	0.986	0.984	0.989
	Length	μ	NA	NA	1.978E+10	4.764	6.873	5.282	7.022	3.835
		σ^2_α	22.435	14.544	∞	21.087	62.072	43.214	83.048	21.404
		σ^2_ξ	NA	NA	0.871	1.166	1.169	1.164	1.167	1.098
		σ^2_β	3.076	2.365	4.808E+42	3.362	3.520	3.036	3.326	2.562
2	Lower	μ	NA	NA	0.894	0.990	0.998	0.990	0.998	0.984
		σ^2_α	0.949	0.932	0.999	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.897	0.964	0.964	0.965	0.964	0.949
		σ^2_β	0.949	0.942	0.916	0.997	0.999	0.999	0.998	0.983
	Upper	μ	NA	NA	0.900	0.991	0.998	0.987	0.998	0.985
		σ^2_α	0.950	0.941	0.969	0.953	0.942	0.954	0.950	0.978
		σ^2_ξ	NA	NA	0.970	0.941	0.941	0.940	0.939	0.954
		σ^2_β	0.934	0.936	0.959	0.935	0.936	0.936	0.936	0.960
	Two	μ	NA	NA	0.846	0.993	1.000	0.991	0.999	0.995
		σ^2_α	0.942	0.931	0.988	0.978	0.978	0.981	0.979	0.992
		σ^2_ξ	NA	NA	0.923	0.950	0.949	0.949	0.951	0.951
		σ^2_β	0.946	0.935	0.965	0.966	0.966	0.966	0.965	0.980
	Length	μ	NA	NA	3.857E+10	4.583	6.741	4.857	6.876	4.320
		σ^2_α	28.209	24.875	∞	18.226	56.335	40.536	82.121	29.963
		σ^2_ξ	NA	NA	0.407	0.557	0.558	0.559	0.558	0.515
		σ^2_β	3.580	3.210	1.105E+17	4.140	4.402	3.676	4.124	3.168
3	Lower	μ	NA	NA	0.925	0.991	0.999	0.983	0.996	0.994
		σ^2_α	0.953	0.927	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.931	0.975	0.973	0.973	0.974	0.960
		σ^2_β	0.952	0.939	0.824	0.971	0.976	0.966	0.973	0.930
	Upper	μ	NA	NA	0.919	0.990	0.997	0.986	0.995	0.991
		σ^2_α	0.958	0.955	0.933	0.951	0.951	0.953	0.960	0.973
		σ^2_ξ	NA	NA	0.959	0.912	0.911	0.910	0.910	0.934
		σ^2_β	0.938	0.932	0.978	0.929	0.926	0.929	0.929	0.959
	Two	μ	NA	NA	0.888	0.993	1.000	0.989	0.999	0.998
		σ^2_α	0.957	0.948	0.966	0.983	0.982	0.981	0.984	0.992
		σ^2_ξ	NA	NA	0.940	0.935	0.936	0.933	0.936	0.937
		σ^2_β	0.944	0.924	0.874	0.943	0.945	0.940	0.941	0.942
	Length	μ	NA	NA	1.529E+11	3.950	6.046	4.112	5.877	4.609
		σ^2_α	24.753	22.496	∞	16.301	50.581	34.157	64.176	32.074
		σ^2_ξ	NA	NA	0.149	0.204	0.203	0.204	0.204	0.184
		σ^2_β	5.976	5.818	5.385E+02	5.958	6.958	5.520	6.431	4.861
4	Lower	μ	NA	NA	0.881	0.987	0.995	0.984	0.995	0.974
		σ^2_α	0.948	0.931	0.995	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.884	0.958	0.960	0.959	0.961	0.941
		σ^2_β	0.940	0.944	1.000	1.000	1.000	1.000	1.000	1.000
	Upper	μ	NA	NA	0.880	0.987	0.998	0.987	0.997	0.979
		σ^2_α	0.942	0.929	0.976	0.942	0.935	0.947	0.936	0.978
		σ^2_ξ	NA	NA	0.977	0.948	0.950	0.950	0.950	0.963
		σ^2_β	0.957	0.944	0.911	0.948	0.946	0.950	0.951	0.971
	Two	μ	NA	NA	0.807	0.992	1.000	0.991	0.999	0.991
		σ^2_α	0.942	0.925	0.989	0.973	0.972	0.981	0.975	0.992
		σ^2_ξ	NA	NA	0.918	0.955	0.958	0.957	0.957	0.949
		σ^2_β	0.947	0.941	0.948	0.979	0.979	0.982	0.980	0.987
	Length	μ	NA	NA	5.218E+11	29.669	43.824	31.826	43.579	28.000
		σ^2_α	1407.000	1264.000	∞	791.249	2464.103	1719.150	3284.250	1454.700
		σ^2_ξ	NA	NA	20.366	27.250	27.276	27.205	27.222	25.500
		σ^2_β	81.291	67.004	2.289E+78	110.751	115.243	99.984	107.876	81.700
5	Lower	μ	NA	NA	0.910	0.995	0.999	0.994	0.998	0.989
		σ^2_α	0.954	0.946	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.904	0.967	0.966	0.967	0.967	0.952
		σ^2_β	0.943	0.937	0.913	0.995	0.998	0.996	0.998	0.979
	Upper	μ	NA	NA	0.895	0.994	0.998	0.993	0.998	0.988
		σ^2_α	0.946	0.940	0.953	0.946	0.940	0.948	0.943	0.982
		σ^2_ξ	NA	NA	0.965	0.925	0.923	0.923	0.925	0.939
		σ^2_β	0.953	0.949	0.971	0.951	0.949	0.952	0.950	0.969
	Two	μ	NA	NA	0.861	0.995	1.000	0.997	1.000	0.997
		σ^2_α	0.949	0.944	0.977	0.976	0.974	0.978	0.976	0.991
		σ^2_ξ	NA	NA	0.921	0.943	0.944	0.945	0.943	0.945
		σ^2_β	0.947	0.941	0.969	0.977	0.978	0.978	0.979	0.981
	Length	μ	NA	NA	1.343E+11	7.961	11.812	8.412	11.969	7.542
		σ^2_α	84.976	71.811	∞	56.606	174.463	127.690	243.480	90.824
		σ^2_ξ	NA	NA	1.379	1.886	1.892	1.888	1.893	1.742
		σ^2_β	11.246	10.126	5.441E+30	12.767	13.738	11.505	12.872	9.844

Simulation results for 95% confidence intervals. Intervals were not determined for **USS** or **TYPEI** for μ or σ^2_ϵ.

TABLE 8
Two-fold nested: model design 3.

Par. design	Type	Par.	USS	TYPEI	HLMM	BAY$1_{1.5}$	BAY1_3	BAY$2_{1.5}$	BAY2_3	FID
1	Lower	μ	NA	NA	0.912	0.998	0.999	0.995	1.000	0.991
		σ^2_α	0.952	0.952	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.838	0.963	0.962	0.965	0.963	0.939
		σ^2_β	0.954	0.954	0.999	1.000	1.000	1.000	1.000	1.000
	Upper	μ	NA	NA	0.916	0.999	1.000	0.998	1.000	0.993
		σ^2_α	0.948	0.948	0.963	0.957	0.951	0.962	0.967	0.977
		σ^2_ξ	NA	NA	0.981	0.957	0.955	0.957	0.954	0.970
		σ^2_β	0.955	0.955	0.820	0.970	0.974	0.973	0.964	0.979
	Two	μ	NA	NA	0.886	1.000	1.000	0.999	1.000	0.997
		σ^2_α	0.952	0.952	0.985	0.980	0.979	0.981	0.991	0.991
		σ^2_ξ	NA	NA	0.884	0.961	0.964	0.963	0.957	0.958
		σ^2_β	0.953	0.953	0.858	0.986	0.991	0.991	0.984	0.990
	Length	μ	NA	NA	8.509E+01	4.175	5.867	4.596	7.379	2.999
		σ^2_α	13.926	13.926	∞	18.592	52.102	34.499	93.757	13.953
		σ^2_ξ	NA	NA	0.992	1.514	1.518	1.514	1.168	1.384
		σ^2_β	1.797	1.797	9.958E+29	2.105	2.126	1.984	3.313	1.598
2	Lower	μ	NA	NA	0.911	0.990	0.995	0.992	0.996	0.982
		σ^2_α	0.958	0.958	0.997	1.000	1.000	1.000	1.000	0.999
		σ^2_ξ	NA	NA	0.890	0.977	0.979	0.980	0.977	0.967
		σ^2_β	0.949	0.949	0.909	0.988	0.991	0.990	0.991	0.965
	Upper	μ	NA	NA	0.894	0.991	0.997	0.991	0.995	0.978
		σ^2_α	0.951	0.951	0.977	0.940	0.933	0.942	0.936	0.970
		σ^2_ξ	NA	NA	0.970	0.932	0.936	0.935	0.937	0.955
		σ^2_β	0.947	0.947	0.949	0.961	0.960	0.960	0.963	0.970
	Two	μ	NA	NA	0.862	0.993	0.999	0.995	0.999	0.988
		σ^2_α	0.955	0.955	0.991	0.970	0.967	0.970	0.966	0.984
		σ^2_ξ	NA	NA	0.920	0.957	0.957	0.958	0.959	0.961
		σ^2_β	0.952	0.952	0.948	0.977	0.983	0.983	0.982	0.982
	Length	μ	NA	NA	1.436E+09	4.279	6.296	4.724	6.417	3.478
		σ^2_α	21.571	21.571	∞	17.762	53.206	44.458	72.727	21.277
		σ^2_ξ	NA	NA	0.511	0.857	0.859	0.859	0.859	0.789
		σ^2_β	1.985	1.985	1.263E+10	2.313	2.345	2.199	2.287	1.755
3	Lower	μ	NA	NA	0.927	0.991	0.998	0.992	0.999	0.992
		σ^2_α	0.941	0.941	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.893	0.974	0.976	0.974	0.974	0.968
		σ^2_β	0.947	0.947	0.845	0.959	0.963	0.957	0.962	0.935
	Upper	μ	NA	NA	0.920	0.992	0.998	0.989	0.998	0.990
		σ^2_α	0.951	0.951	0.949	0.939	0.937	0.943	0.940	0.975
		σ^2_ξ	NA	NA	0.970	0.924	0.922	0.921	0.922	0.940
		σ^2_β	0.947	0.947	0.984	0.937	0.937	0.936	0.939	0.962
	Two	μ	NA	NA	0.901	0.997	1.000	0.996	1.000	0.998
		σ^2_α	0.951	0.951	0.976	0.974	0.975	0.974	0.975	0.988
		σ^2_ξ	NA	NA	0.919	0.947	0.946	0.947	0.947	0.951
		σ^2_β	0.946	0.946	0.891	0.952	0.958	0.953	0.959	0.952
	Length	μ	NA	NA	4.660E+05	3.742	5.698	3.840	5.535	3.374
		σ^2_α	17.089	17.089	∞	16.303	48.044	28.595	60.874	17.326
		σ^2_ξ	NA	NA	0.198	0.374	0.374	0.375	0.374	0.360
		σ^2_β	3.337	3.337	6.063E+21	3.747	3.825	3.466	3.698	2.817
4	Lower	μ	NA	NA	0.890	0.989	0.994	0.987	0.996	0.967
		σ^2_α	0.943	0.943	0.979	1.000	1.000	1.000	1.000	0.982
		σ^2_ξ	NA	NA	0.855	0.967	0.968	0.970	0.969	0.951
		σ^2_β	0.946	0.946	0.998	1.000	1.000	1.000	1.000	1.000
	Upper	μ	NA	NA	0.888	0.990	0.997	0.988	0.992	0.971
		σ^2_α	0.954	0.954	0.987	0.935	0.928	0.937	0.930	0.968
		σ^2_ξ	NA	NA	0.978	0.956	0.957	0.958	0.959	0.972
		σ^2_β	0.953	0.953	0.869	0.974	0.976	0.977	0.976	0.977
	Two	μ	NA	NA	0.825	0.992	1.000	0.995	0.998	0.981
		σ^2_α	0.947	0.947	0.991	0.970	0.969	0.973	0.966	0.986
		σ^2_ξ	NA	NA	0.905	0.966	0.968	0.967	0.967	0.962
		σ^2_β	0.952	0.952	0.907	0.990	0.991	0.990	0.990	0.991
	Length	μ	NA	NA	7.220E+09	28.244	41.182	30.764	42.238	24.000
		σ^2_α	1101.800	1101.800	1.059E+188	765.237	2327.350	1619.508	3212.311	1087.700
		σ^2_ξ	NA	NA	24.058	37.283	37.241	36.375	36.403	34.100
		σ^2_β	49.613	49.613	1.733E+74	65.814	64.843	61.781	63.678	48.800
5	Lower	μ	NA	NA	0.899	0.994	0.998	0.989	0.999	0.978
		σ^2_α	0.940	0.940	1.000	1.000	1.000	1.000	1.000	0.995
		σ^2_ξ	NA	NA	0.890	0.982	0.982	0.983	0.982	0.973
		σ^2_β	0.946	0.946	0.919	0.986	0.989	0.987	0.989	0.965
	Upper	μ	NA	NA	0.910	0.991	0.997	0.989	0.996	0.977
		σ^2_α	0.948	0.948	0.976	0.938	0.933	0.944	0.938	0.966
		σ^2_ξ	NA	NA	0.964	0.924	0.923	0.923	0.923	0.942
		σ^2_β	0.945	0.945	0.947	0.962	0.964	0.966	0.964	0.975
	Two	μ	NA	NA	0.858	0.997	1.000	0.997	1.000	0.992
		σ^2_α	0.939	0.939	0.988	0.967	0.967	0.974	0.967	0.985
		σ^2_ξ	NA	NA	0.909	0.955	0.954	0.956	0.956	0.958
		σ^2_β	0.942	0.942	0.951	0.981	0.982	0.983	0.983	0.980
	Length	μ	NA	NA	4.253E+06	7.456	10.884	8.064	10.939	5.861
		σ^2_α	58.878	58.878	6.153E+192	55.029	162.619	108.797	219.153	58.232
		σ^2_ξ	NA	NA	1.739	2.915	2.925	2.919	2.925	2.684
		σ^2_β	6.757	6.757	6.389E+27	7.638	7.759	7.160	7.548	5.834

Simulation results for 95% confidence intervals. Intervals were not determined for **USS** or **TYPEI** for μ or σ^2_ϵ.

TABLE 9
Two-fold nested: model design 4.

Par. design	Type	Par.	**USS**	**TYPEI**	**HLMM**	**BAY1**$_{1.5}$	**BAY1**$_3$	**BAY2**$_{1.5}$	**BAY2**$_3$	**FID**
1	Lower	μ	NA	NA	0.928	0.986	0.986	0.985	0.987	0.980
		σ^2_α	0.947	0.940	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.830	0.948	0.944	0.947	0.946	0.933
		σ^2_β	0.947	0.947	1.000	1.000	1.000	1.000	1.000	1.000
	Upper	μ	NA	NA	0.924	0.986	0.988	0.988	0.989	0.979
		σ^2_α	0.952	0.955	0.923	0.987	0.989	0.992	0.991	0.995
		σ^2_ξ	NA	NA	0.980	0.964	0.967	0.967	0.967	0.972
		σ^2_β	0.946	0.946	0.812	0.978	0.977	0.979	0.977	0.984
	Two	μ	NA	NA	0.904	0.990	0.993	0.990	0.991	0.985
		σ^2_α	0.948	0.955	0.955	0.997	0.999	0.998	1.000	0.999
		σ^2_ξ	NA	NA	0.873	0.956	0.957	0.957	0.956	0.952
		σ^2_β	0.943	0.943	0.858	0.989	0.991	0.994	0.991	0.993
	Length	μ	NA	NA	1.900E+00	1.883	1.866	1.833	1.850	1.623
		σ^2_α	3.090	2.662	1.058E+46	5.061	5.067	4.453	4.849	2.895
		σ^2_ξ	NA	NA	0.847	1.180	1.181	1.180	1.181	1.090
		σ^2_β	1.797	1.797	2.503E+23	1.331	1.323	1.293	1.318	1.077
2	Lower	μ	NA	NA	0.925	0.975	0.976	0.974	0.976	0.965
		σ^2_α	0.949	0.944	0.991	1.000	1.000	0.999	1.000	0.987
		σ^2_ξ	NA	NA	0.885	0.969	0.972	0.971	0.971	0.968
		σ^2_β	0.954	0.954	0.887	0.985	0.985	0.986	0.986	0.968
	Upper	μ	NA	NA	0.923	0.973	0.975	0.968	0.972	0.962
		σ^2_α	0.954	0.946	0.938	0.966	0.965	0.965	0.966	0.973
		σ^2_ξ	NA	NA	0.949	0.917	0.915	0.916	0.916	0.915
		σ^2_β	0.952	0.952	0.945	0.968	0.970	0.972	0.971	0.982
	Two	μ	NA	NA	0.899	0.977	0.977	0.976	0.978	0.968
		σ^2_α	0.947	0.944	0.966	0.988	0.986	0.990	0.988	0.988
		σ^2_ξ	NA	NA	0.903	0.940	0.941	0.938	0.940	0.941
		σ^2_β	0.948	0.948	0.938	0.983	0.984	0.984	0.985	0.984
	Length	μ	NA	NA	1.339E+00	2.110	2.142	2.063	2.098	1.804
		σ^2_α	4.282	3.830	7.593E+45	6.565	7.066	5.869	6.589	3.980
		σ^2_ξ	NA	NA	0.451	0.691	0.694	0.693	0.693	0.660
		σ^2_β	2.010	2.010	8.048E+02	1.617	1.613	1.571	1.602	1.309
3	Lower	μ	NA	NA	0.929	0.975	0.981	0.978	0.982	0.966
		σ^2_α	0.947	0.947	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.909	0.972	0.973	0.972	0.971	0.973
		σ^2_β	0.951	0.951	0.812	0.957	0.960	0.957	0.956	0.916
	Upper	μ	NA	NA	0.933	0.973	0.979	0.976	0.980	0.966
		σ^2_α	0.944	0.947	0.828	0.952	0.959	0.959	0.960	0.962
		σ^2_ξ	NA	NA	0.970	0.926	0.921	0.922	0.924	0.861
		σ^2_β	0.950	0.950	0.978	0.963	0.965	0.961	0.964	0.969
	Two	μ	NA	NA	0.910	0.979	0.987	0.980	0.986	0.973
		σ^2_α	0.941	0.951	0.878	0.977	0.980	0.982	0.985	0.983
		σ^2_ξ	NA	NA	0.931	0.952	0.950	0.950	0.950	0.896
		σ^2_β	0.947	0.947	0.862	0.962	0.959	0.960	0.960	0.946
	Length	μ	NA	NA	1.646E+00	2.208	2.370	2.153	2.250	1.905
		σ^2_α	4.427	3.734	1.088E+57	6.869	8.268	6.076	7.069	3.939
		σ^2_ξ	NA	NA	0.171	0.274	0.274	0.275	0.275	0.309
		σ^2_β	3.284	3.284	1.543E+00	2.401	2.408	2.326	2.378	1.952
4	Lower	μ	NA	NA	0.910	0.969	0.972	0.971	0.969	0.956
		σ^2_α	0.946	0.944	0.933	0.996	0.998	0.997	0.997	0.963
		σ^2_ξ	NA	NA	0.865	0.966	0.966	0.965	0.966	0.950
		σ^2_β	0.951	0.951	0.999	1.000	1.000	1.000	1.000	1.000
	Upper	μ	NA	NA	0.910	0.971	0.976	0.966	0.969	0.958
		σ^2_α	0.962	0.943	0.971	0.974	0.974	0.976	0.978	0.983
		σ^2_ξ	NA	NA	0.973	0.952	0.953	0.953	0.953	0.958
		σ^2_β	0.954	0.954	0.849	0.970	0.975	0.975	0.974	0.980
	Two	μ	NA	NA	0.879	0.978	0.983	0.979	0.980	0.965
		σ^2_α	0.955	0.947	0.979	0.988	0.990	0.989	0.989	0.986
		σ^2_ξ	NA	NA	0.899	0.957	0.960	0.961	0.958	0.956
		σ^2_β	0.953	0.953	0.894	0.988	0.991	0.990	0.991	0.994
	Length	μ	NA	NA	3.873E+16	13.938	14.028	13.457	13.772	11.766
		σ^2_α	192.588	182.357	∞	294.096	314.870	261.466	294.830	180.137
		σ^2_ξ	NA	NA	21.383	30.757	30.776	30.695	30.659	28.330
		σ^2_β	50.088	50.088	7.257E+20	48.410	48.592	47.432	48.217	39.314
5	Lower	μ	NA	NA	0.921	0.978	0.979	0.975	0.977	0.963
		σ^2_α	0.942	0.939	0.997	1.000	1.000	1.000	1.000	0.994
		σ^2_ξ	NA	NA	0.883	0.980	0.979	0.979	0.980	0.973
		σ^2_β	0.952	0.952	0.870	0.979	0.979	0.981	0.982	0.958
	Upper	μ	NA	NA	0.915	0.977	0.981	0.978	0.976	0.968
		σ^2_α	0.955	0.949	0.915	0.968	0.966	0.969	0.967	0.976
		σ^2_ξ	NA	NA	0.964	0.933	0.931	0.930	0.931	0.930
		σ^2_β	0.951	0.951	0.949	0.973	0.975	0.976	0.974	0.983
	Two	μ	NA	NA	0.894	0.983	0.988	0.985	0.987	0.972
		σ^2_α	0.949	0.943	0.953	0.987	0.988	0.987	0.990	0.990
		σ^2_ξ	NA	NA	0.911	0.959	0.960	0.962	0.961	0.952
		σ^2_β	0.948	0.948	0.928	0.984	0.986	0.985	0.984	0.983
	Length	μ	NA	NA	1.294E+06	3.680	3.692	3.596	3.607	3.105
		σ^2_α	12.485	11.067	∞	19.750	20.945	17.667	19.339	11.641
		σ^2_ξ	NA	NA	1.469	2.260	2.260	2.267	2.263	2.153
		σ^2_β	6.613	6.613	1.285E+05	5.079	5.106	4.977	5.061	4.117

Simulation results for 95% confidence intervals. Intervals were not determined for **USS** or **TYPEI** for μ or σ^2_ϵ.

TABLE 10
Two-fold nested: model design 5.

Par. design	Type	Par.	USS	TYPEI	HLMM	BAY1$_{1.5}$	BAY1$_3$	BAY2$_{1.5}$	BAY2$_3$	FID
1	Lower	μ	NA	NA	0.914	0.998	1.000	0.999	0.999	0.998
		σ^2_α	0.948	0.938	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.813	0.961	0.962	0.961	0.961	0.930
		σ^2_β	0.953	0.953	1.000	1.000	1.000	1.000	1.000	1.000
	Upper	μ	NA	NA	0.910	0.999	1.000	0.997	0.999	0.995
		σ^2_α	0.958	0.966	0.945	0.984	0.985	0.991	0.985	0.997
		σ^2_ξ	NA	NA	0.989	0.959	0.958	0.956	0.958	0.976
		σ^2_β	0.948	0.947	0.827	0.960	0.962	0.969	0.963	0.978
	Two	μ	NA	NA	0.875	0.999	1.000	0.999	1.000	0.999
		σ^2_α	0.953	0.959	0.971	0.998	0.998	0.998	0.997	1.000
		σ^2_ξ	NA	NA	0.861	0.965	0.966	0.968	0.969	0.957
		σ^2_β	0.948	0.951	0.862	0.984	0.988	0.988	0.989	0.991
	Length	μ	NA	NA	1.023E+10	4.765	6.897	5.532	7.501	4.320
		σ^2_α	20.816	18.702	∞	16.721	53.458	40.403	86.028	22.869
		σ^2_ξ	NA	NA	1.198	2.363	2.402	2.350	2.387	1.922
		σ^2_β	9.026	8.561	2.832E+52	7.955	10.414	7.123	9.236	5.223
2	Lower	μ	NA	NA	0.908	0.993	0.999	0.994	0.998	0.998
		σ^2_α	0.945	0.943	0.998	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.861	0.976	0.973	0.974	0.975	0.955
		σ^2_β	0.934	0.935	0.990	1.000	0.999	0.999	0.999	0.996
	Upper	μ	NA	NA	0.902	0.992	0.998	0.991	0.998	0.987
		σ^2_α	0.957	0.961	0.973	0.984	0.977	0.986	0.981	0.990
		σ^2_ξ	NA	NA	0.977	0.941	0.942	0.940	0.941	0.959
		σ^2_β	0.953	0.954	0.952	0.973	0.973	0.977	0.977	0.979
	Two	μ	NA	NA	0.857	0.995	1.000	0.994	1.000	0.995
		σ^2_α	0.947	0.956	0.990	0.996	0.996	0.998	0.996	0.998
		σ^2_ξ	NA	NA	0.893	0.960	0.961	0.963	0.962	0.957
		σ^2_β	0.940	0.941	0.968	0.988	0.986	0.988	0.986	0.990
	Length	μ	NA	NA	7.191E+07	4.491	6.573	5.138	7.099	4.305
		σ^2_α	26.201	26.148	∞	14.493	47.272	40.848	76.186	27.388
		σ^2_ξ	NA	NA	1567.622	1.249	1.260	1.249	1.259	0.994
		σ^2_β	7.339	7.113	2.703E+42	6.717	8.827	6.119	7.760	4.507
3	Lower	μ	NA	NA	0.927	0.992	0.999	0.990	0.998	0.992
		σ^2_α	0.947	0.946	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.886	0.981	0.982	0.982	0.981	0.963
		σ^2_β	0.951	0.950	0.839	0.986	0.988	0.979	0.988	0.958
	Upper	μ	NA	NA	0.911	0.993	0.998	0.994	1.000	0.994
		σ^2_α	0.963	0.972	0.923	0.964	0.959	0.963	0.959	0.979
		σ^2_ξ	NA	NA	0.965	0.911	0.909	0.912	0.909	0.942
		σ^2_β	0.946	0.946	0.983	0.955	0.953	0.961	0.956	0.974
	Two	μ	NA	NA	0.898	0.994	1.000	0.995	1.000	0.998
		σ^2_α	0.953	0.957	0.963	0.983	0.987	0.987	0.985	0.992
		σ^2_ξ	NA	NA	0.903	0.944	0.945	0.943	0.945	0.950
		σ^2_β	0.951	0.943	0.908	0.977	0.980	0.977	0.980	0.967
	Length	μ	NA	NA	5.400E+07	4.237	6.430	4.683	6.681	4.338
		σ^2_α	22.608	26.284	∞	13.773	44.965	34.665	72.024	25.011
		σ^2_ξ	NA	NA	0.403	0.601	0.597	0.604	0.597	0.461
		σ^2_β	10.947	10.821	∞	7.947	11.428	7.845	10.203	6.108
4	Lower	μ	NA	NA	0.886	0.991	0.997	0.989	0.998	0.984
		σ^2_α	0.939	0.933	0.993	1.000	1.000	1.000	1.000	0.998
		σ^2_ξ	NA	NA	0.840	0.965	0.965	0.963	0.964	0.934
		σ^2_β	0.942	0.940	1.000	1.000	1.000	1.000	1.000	1.000
	Upper	μ	NA	NA	0.872	0.992	1.000	0.991	0.998	0.986
		σ^2_α	0.953	0.964	0.984	0.983	0.980	0.981	0.978	0.990
		σ^2_ξ	NA	NA	0.985	0.951	0.952	0.954	0.953	0.970
		σ^2_β	0.956	0.956	0.861	0.964	0.968	0.972	0.971	0.978
	Two	μ	NA	NA	0.815	0.995	1.000	0.995	1.000	0.995
		σ^2_α	0.946	0.951	0.995	0.994	0.994	0.996	0.997	0.999
		σ^2_ξ	NA	NA	0.879	0.960	0.960	0.961	0.962	0.953
		σ^2_β	0.944	0.950	0.900	0.986	0.986	0.986	0.985	0.986
	Length	μ	NA	NA	5.180E+10	29.274	42.439	33.836	44.578	28.700
		σ^2_α	1301.100	1248.300	∞	629.322	2038.176	1623.013	3068.150	1303.100
		σ^2_ξ	NA	NA	29.998	59.347	60.068	50.015	50.295	47.500
		σ^2_β	224.912	213.054	1.895E+110	255.056	321.920	230.582	282.787	159.300
5	Lower	μ	NA	NA	0.901	0.996	1.000	0.996	1.000	0.994
		σ^2_α	0.957	0.944	0.998	1.000	1.000	1.000	1.000	1.000
		σ^2_ξ	NA	NA	0.847	0.973	0.971	0.971	0.973	0.945
		σ^2_β	0.954	0.954	0.985	1.000	1.000	1.000	1.000	0.997
	Upper	μ	NA	NA	0.903	0.994	0.999	0.996	0.999	0.991
		σ^2_α	0.958	0.964	0.962	0.980	0.977	0.983	0.978	0.989
		σ^2_ξ	NA	NA	0.979	0.938	0.938	0.938	0.938	0.967
		σ^2_β	0.953	0.952	0.952	0.974	0.973	0.976	0.973	0.980
	Two	μ	NA	NA	0.860	0.997	1.000	0.997	1.000	0.998
		σ^2_α	0.960	0.956	0.986	0.996	0.995	0.995	0.996	0.999
		σ^2_ξ	NA	NA	0.882	0.962	0.962	0.959	0.960	0.958
		σ^2_β	0.954	0.958	0.973	0.987	0.986	0.989	0.987	0.991
	Length	μ	NA	NA	8.653E+10	7.846	11.522	8.963	12.292	7.491
		σ^2_α	76.309	76.195	∞	44.903	146.087	109.324	230.460	80.010
		σ^2_ξ	NA	NA	2.048	4.149	4.191	4.139	4.176	3.312
		σ^2_β	24.305	23.698	2.196E+49	21.382	28.213	19.242	24.958	14.519

Simulation results for 95% confidence intervals. Intervals were not determined for **USS** or **TYPEI** for μ or σ^2_ϵ.

3. Two-factor crossed with interaction. Box plot summaries for each non-error variance component for the two-factor crossed with interaction model of Section 3.2 are displayed in this section. In addition, the complete results for the simulation are included. Intervals were not determined for **HB** for μ or σ_ϵ^2 because the corresponding paper did not propose a methodology to compute them. We also note that not all 2000 data sets converged for **HLMM**; the empirical coverage and average lengths for **HLMM** were computed only using the converging data sets. The percentage of converging data sets is displayed in Table 11.

TABLE 11
Two-factor crossed with interaction: percentage of converged data sets for **HLMM** *(out of 2000 data sets).*

	Parameter design				
Model Design	1	2	3	4	5
1	99.65	98.65	98.60	99.35	99.65
2	99.45	98.40	97.25	98.70	99.20
3	99.35	98.65	98.20	99.20	99.10
4	98.35	96.70	95.20	97.80	98.25
5	99.75	99.20	99.15	99.70	99.85
6	99.85	99.45	98.55	99.85	100.00

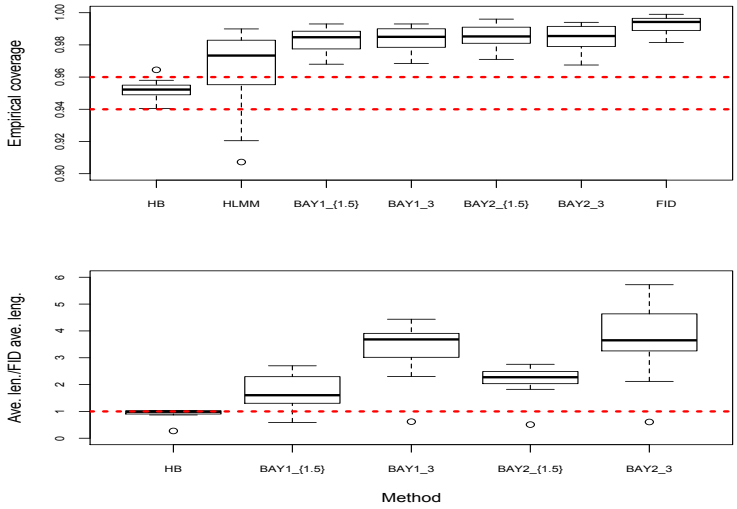

FIG 3. *Simulation results for 95% two-sided confidence intervals on σ_α^2 the two-factor crossed with interaction model of Equation (3.2). The top plot is of the empirical coverage probabilities of the intervals, and the bottom plot is of the average interval lengths divided by the average interval lengths of* **FID**. *Average interval lengths are not included for* **HLMM** *because of their excessive lengths would hinder the scale of the plot.*

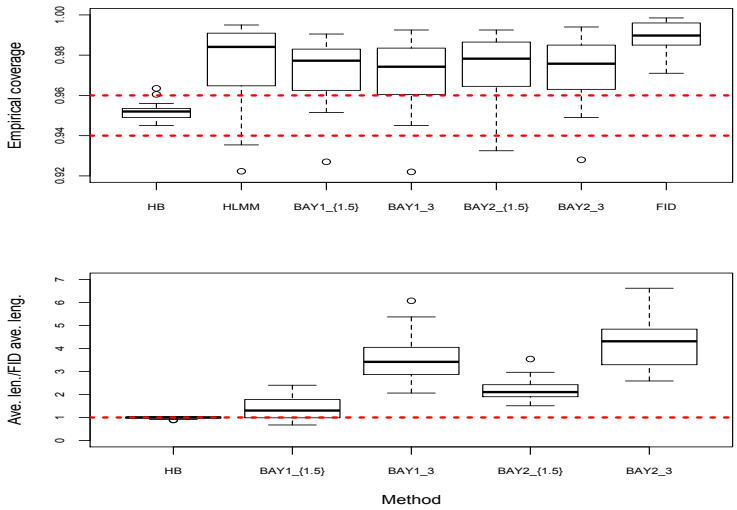

FIG 4. *Simulation results for 95% two-sided confidence intervals on σ_β^2 the two-factor crossed with interaction model of Equation (3.2). The top plot is of the empirical coverage probabilities of the intervals, and the bottom plot is of the average interval lengths divided by the average interval lengths of* **FID**. *Average interval lengths are not included for* **HLMM** *because of their excessive lengths would hinder the scale of the plot.*

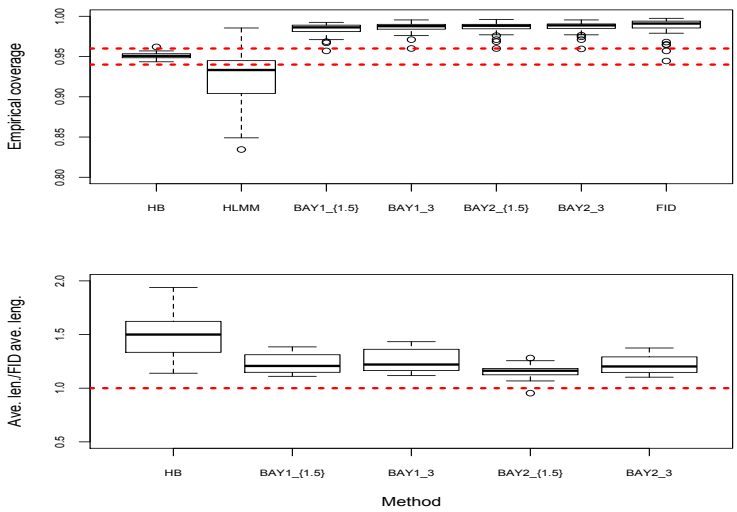

FIG 5. *Simulation results for 95% two-sided confidence intervals on* $\sigma^2_{\alpha\beta}$ *the two-factor crossed with interaction model of Equation (3.2). The top plot is of the empirical coverage probabilities of the intervals, and the bottom plot is of the average interval lengths divided by the average interval lengths of* **FID**. *Average interval lengths are not included for* **HLMM** *because of their excessive lengths would hinder the scale of the plot.*

TABLE 12
Two-factor crossed with interaction: model design 1.

Par. design	Type	Par.	**HB**	**HLMM**	**BAY1**$_{1.5}$	**BAY1**$_{3}$	**BAY2**$_{1.5}$	**BAY2**$_{3}$	**FID**
1	Lower	μ	NA	0.915	0.995	0.998	0.994	0.998	0.983
		σ^2_α	0.951	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.946	0.994	1.000	1.000	1.000	1.000	0.998
		σ^2_β	0.942	0.964	1.000	1.000	1.000	1.000	0.978
		σ^2_ϵ	NA	0.863	0.964	0.966	0.964	0.964	0.950
	Upper	μ	NA	0.892	0.991	0.996	0.988	0.998	0.981
		σ^2_α	0.956	0.951	0.972	0.973	0.977	0.975	0.988
		$\sigma^2_{\alpha\beta}$	0.955	0.900	0.975	0.974	0.974	0.973	0.978
		σ^2_β	0.956	0.992	0.924	0.914	0.925	0.915	0.967
		σ^2_ϵ	NA	0.969	0.949	0.947	0.949	0.947	0.961
	Two	μ	NA	0.861	0.994	0.998	0.994	0.998	0.989
		σ^2_α	0.957	0.978	0.992	0.990	0.993	0.992	0.995
		$\sigma^2_{\alpha\beta}$	0.950	0.932	0.987	0.987	0.985	0.986	0.988
		σ^2_β	0.949	0.991	0.958	0.957	0.959	0.958	0.987
		σ^2_ϵ	NA	0.893	0.957	0.957	0.957	0.957	0.953
	Length	μ	NA	5.222E+10	4.556	6.619	4.757	6.704	3.718
		σ^2_α	2.677	6.370E+54	6.811	10.173	6.357	8.828	2.618
		$\sigma^2_{\alpha\beta}$	1.199	7.413E+16	1.103	1.098	1.068	1.089	0.899
		σ^2_β	22.109	2.430E+106	17.863	53.399	35.357	73.201	21.611
		σ^2_ϵ	NA	0.423	0.634	0.635	0.635	0.635	0.586
2	Lower	μ	NA	0.923	0.996	0.999	0.998	0.999	0.988
		σ^2_α	0.953	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.951	0.999	1.000	1.000	1.000	1.000	1.000
		σ^2_β	0.958	0.998	1.000	1.000	1.000	1.000	0.998
		σ^2_ϵ	NA	0.850	0.961	0.963	0.961	0.963	0.943
	Upper	μ	NA	0.915	0.996	1.000	0.997	1.000	0.990
		σ^2_α	0.952	0.937	0.980	0.979	0.980	0.979	0.988
		$\sigma^2_{\alpha\beta}$	0.960	0.872	0.981	0.985	0.986	0.985	0.988
		σ^2_β	0.943	0.982	0.926	0.922	0.933	0.927	0.963
		σ^2_ϵ	NA	0.974	0.958	0.957	0.959	0.956	0.966
	Two	μ	NA	0.893	0.999	1.000	0.998	1.000	0.996
		σ^2_α	0.952	0.969	0.993	0.993	0.993	0.994	0.997
		$\sigma^2_{\alpha\beta}$	0.954	0.906	0.992	0.996	0.996	0.996	0.997
		σ^2_β	0.949	0.993	0.962	0.958	0.965	0.960	0.987
		σ^2_ϵ	NA	0.883	0.962	0.963	0.961	0.963	0.963
	Length	μ	NA	8.744E+10	4.690	6.609	4.905	6.622	3.418
		σ^2_α	3.239	7.780E+55	8.268	12.316	7.939	11.291	3.163
		$\sigma^2_{\alpha\beta}$	1.677	6.650E+21	1.421	1.421	1.367	1.407	1.155
		σ^2_β	16.464	1.210E+157	18.939	53.612	33.023	68.413	15.810
		σ^2_ϵ	NA	0.663	0.980	0.984	0.984	0.983	0.911
3	Lower	μ	NA	0.939	0.999	1.000	1.000	1.000	0.998
		σ^2_α	0.946	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.949	0.946	0.997	0.999	0.998	0.999	0.990
		σ^2_β	0.944	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.871	0.966	0.969	0.968	0.969	0.960
	Upper	μ	NA	0.935	1.000	1.000	1.000	1.000	0.996
		σ^2_α	0.953	0.897	0.967	0.966	0.970	0.969	0.986
		$\sigma^2_{\alpha\beta}$	0.954	0.944	0.978	0.979	0.978	0.980	0.982
		σ^2_β	0.948	0.937	0.938	0.935	0.944	0.937	0.976
		σ^2_ϵ	NA	0.961	0.942	0.943	0.941	0.941	0.961
	Two	μ	NA	0.920	1.000	1.000	1.000	1.000	1.000
		σ^2_α	0.949	0.940	0.986	0.989	0.989	0.989	0.994
		$\sigma^2_{\alpha\beta}$	0.953	0.964	0.989	0.991	0.990	0.989	0.991
		σ^2_β	0.951	0.973	0.978	0.971	0.976	0.975	0.992
		σ^2_ϵ	NA	0.900	0.959	0.962	0.962	0.961	0.962
	Length	μ	NA	1.193E+11	4.545	6.311	4.753	6.528	3.180
		σ^2_α	4.130	3.540E+72	9.926	15.204	9.656	14.015	4.118
		$\sigma^2_{\alpha\beta}$	2.650	5.230E+16	1.969	1.977	1.887	1.958	1.615
		σ^2_β	10.242	4.170E+122	18.146	48.741	28.641	66.280	10.201
		σ^2_ϵ	NA	0.717	1.104	1.107	1.105	1.107	1.024

TABLE 13
Two-factor crossed with interaction: model design 1 continued.

Par. design	Type	Par.	**HB**	**HLMM**	**BAY1**$_{1.5}$	**BAY1**$_3$	**BAY2**$_{1.5}$	**BAY2**$_3$	**FID**
4	Lower	μ	NA	0.944	0.999	1.000	0.998	1.000	0.997
		σ^2_α	0.944	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.954	0.768	0.963	0.966	0.960	0.963	0.938
		σ^2_β	0.945	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.887	0.979	0.977	0.975	0.978	0.970
	Upper	μ	NA	0.938	0.998	0.999	0.997	1.000	0.995
		σ^2_α	0.952	0.886	0.949	0.948	0.953	0.952	0.970
		$\sigma^2_{\alpha\beta}$	0.954	0.971	0.973	0.973	0.974	0.974	0.980
		σ^2_β	0.955	0.933	0.939	0.926	0.940	0.936	0.968
		σ^2_ϵ	NA	0.954	0.923	0.921	0.922	0.921	0.940
	Two	μ	NA	0.927	1.000	1.000	0.999	1.000	0.999
		σ^2_α	0.946	0.920	0.974	0.976	0.978	0.977	0.985
		$\sigma^2_{\alpha\beta}$	0.953	0.873	0.973	0.976	0.976	0.977	0.965
		σ^2_β	0.955	0.965	0.968	0.971	0.974	0.971	0.988
		σ^2_ϵ	NA	0.903	0.948	0.952	0.950	0.949	0.950
	Length	μ	NA	3.572E+11	4.480	6.341	4.598	6.153	3.254
		σ^2_α	4.355	7.340E+81	10.193	16.373	10.174	14.604	4.441
		$\sigma^2_{\alpha\beta}$	3.046	3.814E+06	2.286	2.305	2.186	2.273	1.882
		σ^2_β	10.546	1.070E+121	17.608	47.890	28.699	55.458	10.611
		σ^2_ϵ	NA	0.488	0.810	0.813	0.812	0.812	0.758
5	Lower	μ	NA	0.921	0.997	0.999	0.995	0.998	0.990
		σ^2_α	0.938	0.996	1.000	1.000	1.000	1.000	0.987
		$\sigma^2_{\alpha\beta}$	0.952	0.881	0.989	0.991	0.989	0.991	0.978
		σ^2_β	0.956	0.999	1.000	1.000	1.000	1.000	0.999
		σ^2_ϵ	NA	0.878	0.978	0.977	0.979	0.979	0.970
	Upper	μ	NA	0.920	0.995	0.998	0.992	0.999	0.988
		σ^2_α	0.946	0.964	0.948	0.948	0.953	0.948	0.972
		$\sigma^2_{\alpha\beta}$	0.951	0.953	0.972	0.973	0.971	0.973	0.974
		σ^2_β	0.949	0.979	0.930	0.919	0.935	0.925	0.965
		σ^2_ϵ	NA	0.946	0.919	0.919	0.917	0.918	0.942
	Two	μ	NA	0.895	0.998	1.000	0.996	1.000	0.995
		σ^2_α	0.943	0.983	0.978	0.977	0.977	0.979	0.987
		$\sigma^2_{\alpha\beta}$	0.952	0.939	0.990	0.988	0.988	0.990	0.989
		σ^2_β	0.952	0.991	0.964	0.961	0.967	0.965	0.983
		σ^2_ϵ	NA	0.903	0.951	0.954	0.957	0.955	0.962
	Length	μ	NA	3.944E+11	9.004	12.969	9.227	12.713	6.954
		σ^2_α	21.108	4.010E+82	41.376	71.191	42.783	62.984	20.551
		$\sigma^2_{\alpha\beta}$	7.180	3.570E+11	6.507	6.529	6.259	6.446	5.382
		σ^2_β	56.313	1.360E+122	68.650	193.542	115.561	239.728	54.669
		σ^2_ϵ	NA	1.580	2.532	2.536	2.535	2.538	2.357

TABLE 14
Two-factor crossed with interaction: model design 2.

Par. design	Type	Par.	**HB**	**HLMM**	**BAY1**$_{1.5}$	**BAY1**$_3$	**BAY2**$_{1.5}$	**BAY2**$_3$	**FID**
1	Lower	μ	NA	0.907	0.998	0.999	0.993	0.999	0.990
		σ^2_α	0.949	0.999	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.942	0.996	1.000	1.000	1.000	1.000	1.000
		σ^2_β	0.949	0.983	1.000	1.000	1.000	1.000	0.994
		σ^2_ϵ	NA	0.850	0.962	0.964	0.965	0.962	0.943
	Upper	μ	NA	0.906	0.998	0.999	0.997	1.000	0.991
		σ^2_α	0.949	0.960	0.963	0.962	0.967	0.964	0.988
		$\sigma^2_{\alpha\beta}$	0.952	0.909	0.977	0.977	0.980	0.978	0.987
		σ^2_β	0.950	0.986	0.946	0.936	0.946	0.941	0.980
		σ^2_ϵ	NA	0.976	0.948	0.947	0.949	0.948	0.964
	Two	μ	NA	0.859	1.000	1.000	0.999	1.000	0.998
		σ^2_α	0.950	0.983	0.988	0.988	0.988	0.988	0.995
		$\sigma^2_{\alpha\beta}$	0.948	0.941	0.992	0.993	0.993	0.994	0.995
		σ^2_β	0.953	0.992	0.979	0.975	0.981	0.978	0.993
		σ^2_ϵ	NA	0.885	0.956	0.957	0.957	0.956	0.956
	Length	μ	NA	2.284E+11	5.106	7.755	5.651	8.105	4.503
		σ^2_α	8.326	∞	12.952	36.112	20.930	46.587	8.141
		$\sigma^2_{\alpha\beta}$	2.418	1.120E+24	2.177	2.251	2.014	2.157	1.572
		σ^2_β	23.035	∞	15.963	48.889	35.780	68.947	23.716
		σ^2_ϵ	NA	0.464	0.747	0.747	0.745	0.746	0.674
2	Lower	μ	NA	0.911	1.000	1.000	0.999	1.000	0.993
		σ^2_α	0.940	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.957	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_β	0.945	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.839	0.951	0.953	0.951	0.951	0.932
	Upper	μ	NA	0.916	1.000	1.000	0.999	1.000	0.998
		σ^2_α	0.957	0.953	0.970	0.971	0.978	0.969	0.993
		$\sigma^2_{\alpha\beta}$	0.944	0.887	0.973	0.975	0.977	0.976	0.985
		σ^2_β	0.955	0.982	0.964	0.955	0.968	0.961	0.985
		σ^2_ϵ	NA	0.972	0.952	0.952	0.952	0.952	0.972
	Two	μ	NA	0.880	1.000	1.000	1.000	1.000	1.000
		σ^2_α	0.941	0.982	0.992	0.992	0.992	0.994	0.999
		$\sigma^2_{\alpha\beta}$	0.948	0.913	0.985	0.989	0.989	0.990	0.991
		σ^2_β	0.950	0.990	0.986	0.984	0.989	0.985	0.995
		σ^2_ϵ	NA	0.874	0.952	0.951	0.953	0.954	0.953
	Length	μ	NA	6.573E+10	5.410	8.080	6.162	8.727	4.459
		σ^2_α	10.248	∞	15.504	42.716	27.558	55.431	10.000
		$\sigma^2_{\alpha\beta}$	3.560	1.190E+29	2.787	2.914	2.547	2.797	2.035
		σ^2_β	17.315	∞	17.173	50.365	34.539	71.907	17.429
		σ^2_ϵ	NA	0.726	1.153	1.159	1.149	1.157	1.046
3	Lower	μ	NA	0.932	1.000	1.000	0.999	1.000	1.000
		σ^2_α	0.936	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.943	0.986	1.000	1.000	1.000	1.000	0.999
		σ^2_β	0.956	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.870	0.973	0.974	0.952	0.975	0.959
	Upper	μ	NA	0.937	1.000	1.000	0.999	1.000	1.000
		σ^2_α	0.957	0.916	0.963	0.957	0.972	0.960	0.990
		$\sigma^2_{\alpha\beta}$	0.961	0.953	0.981	0.980	0.977	0.981	0.987
		σ^2_β	0.944	0.908	0.954	0.944	0.969	0.952	0.985
		σ^2_ϵ	NA	0.958	0.940	0.939	0.953	0.940	0.960
	Two	μ	NA	0.920	1.000	1.000	1.000	1.000	1.000
		σ^2_α	0.948	0.955	0.987	0.985	0.993	0.988	0.996
		$\sigma^2_{\alpha\beta}$	0.947	0.970	0.991	0.990	0.989	0.991	0.994
		σ^2_β	0.945	0.951	0.987	0.984	0.989	0.985	0.996
		σ^2_ϵ	NA	0.902	0.958	0.959	0.955	0.960	0.962
	Length	μ	NA	1.712E+10	5.666	8.488	6.096	8.966	4.554
		σ^2_α	12.595	∞	17.775	50.477	29.732	69.446	12.915
		$\sigma^2_{\alpha\beta}$	5.200	2.130E+29	3.577	3.783	2.557	3.612	2.682
		σ^2_β	13.087	∞	18.042	51.528	35.018	65.847	14.109
		σ^2_ϵ	NA	0.778	1.266	1.275	1.149	1.273	1.149

TABLE 15
Two-factor crossed with interaction: model design 2 continued.

Par. design	Type	Par.	**HB**	**HLMM**	**BAY**$1_{1.5}$	**BAY**1_3	**BAY**$2_{1.5}$	**BAY**2_3	**FID**
4	Lower	μ	NA	0.936	0.999	1.000	0.998	1.000	0.998
		σ^2_α	0.952	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.957	0.818	0.988	0.993	0.990	0.992	0.967
		σ^2_β	0.947	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.884	0.974	0.975	0.977	0.974	0.969
	Upper	μ	NA	0.939	1.000	1.000	0.998	1.000	0.998
		σ^2_α	0.958	0.907	0.938	0.936	0.946	0.940	0.978
		$\sigma^2_{\alpha\beta}$	0.945	0.978	0.971	0.972	0.973	0.973	0.986
		σ^2_β	0.960	0.913	0.945	0.940	0.952	0.940	0.981
		σ^2_ϵ	NA	0.959	0.921	0.922	0.922	0.921	0.943
	Two	μ	NA	0.917	1.000	1.000	0.998	1.000	0.999
		σ^2_α	0.958	0.946	0.976	0.973	0.982	0.978	0.994
		$\sigma^2_{\alpha\beta}$	0.953	0.927	0.987	0.988	0.990	0.989	0.988
		σ^2_β	0.954	0.949	0.980	0.979	0.983	0.980	0.992
		σ^2_ϵ	NA	0.909	0.949	0.950	0.951	0.951	0.961
	Length	μ	NA	5.548E+10	5.524	8.303	5.934	8.473	4.626
		σ^2_α	13.417	7.970E+190	17.356	49.362	31.632	61.872	14.326
		$\sigma^2_{\alpha\beta}$	5.720	6.390E+20	3.959	4.156	3.612	3.987	2.955
		σ^2_β	13.084	5.920E+118	17.205	49.101	33.604	64.219	13.857
		σ^2_ϵ	NA	0.517	0.881	0.883	0.880	0.882	0.803
5	Lower	μ	NA	0.915	0.997	1.000	0.998	1.000	0.994
		σ^2_α	0.941	0.998	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.942	0.938	0.997	0.999	0.998	0.999	0.990
		σ^2_β	0.946	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.870	0.970	0.971	0.969	0.970	0.959
	Upper	μ	NA	0.906	0.995	1.000	0.996	0.999	0.990
		σ^2_α	0.960	0.978	0.959	0.958	0.963	0.955	0.985
		$\sigma^2_{\alpha\beta}$	0.960	0.958	0.974	0.972	0.975	0.976	0.986
		σ^2_β	0.951	0.972	0.951	0.948	0.956	0.952	0.976
		σ^2_ϵ	NA	0.971	0.927	0.926	0.925	0.926	0.944
	Two	μ	NA	0.872	0.998	1.000	0.998	1.000	0.997
		σ^2_α	0.947	0.990	0.986	0.987	0.988	0.986	0.996
		$\sigma^2_{\alpha\beta}$	0.951	0.967	0.989	0.989	0.990	0.991	0.994
		σ^2_β	0.951	0.991	0.980	0.979	0.984	0.977	0.992
		σ^2_ϵ	NA	0.890	0.948	0.947	0.946	0.945	0.951
	Length	μ	NA	8.791E+09	10.442	15.777	11.438	16.115	9.197
		σ^2_α	59.958	4.010E+122	61.400	176.642	116.957	241.856	58.569
		$\sigma^2_{\alpha\beta}$	13.355	8.620E+39	11.226	11.795	10.337	11.186	8.229
		σ^2_β	62.019	1.980E+133	61.949	180.837	121.127	231.522	63.559
		σ^2_ϵ	NA	1.694	2.798	2.812	2.797	2.807	2.540

TABLE 16
Two-factor crossed with interaction: model design 3.

Par. design	Type	Par.	**HB**	**HLMM**	**BAY1**$_{1.5}$	**BAY1**$_{3}$	**BAY2**$_{1.5}$	**BAY2**$_{3}$	**FID**
1	Lower	μ	NA	0.891	0.996	0.998	0.996	0.999	0.989
		σ^2_α	0.950	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.956	0.998	1.000	1.000	1.000	1.000	1.000
		σ^2_β	0.958	0.971	1.000	1.000	1.000	1.000	0.996
		σ^2_ϵ	NA	0.848	0.962	0.964	0.965	0.963	0.948
	Upper	μ	NA	0.894	0.995	1.000	0.995	1.000	0.990
		σ^2_α	0.950	0.958	0.971	0.969	0.974	0.970	0.991
		$\sigma^2_{\alpha\beta}$	0.947	0.911	0.967	0.967	0.969	0.967	0.979
		σ^2_β	0.958	0.986	0.951	0.946	0.953	0.951	0.982
		σ^2_ϵ	NA	0.968	0.942	0.943	0.943	0.942	0.962
	Two	μ	NA	0.845	0.997	1.000	0.998	1.000	0.997
		σ^2_α	0.953	0.981	0.989	0.991	0.991	0.992	0.998
		$\sigma^2_{\alpha\beta}$	0.955	0.933	0.981	0.984	0.985	0.985	0.991
		σ^2_β	0.964	0.990	0.978	0.977	0.981	0.978	0.994
		σ^2_ϵ	NA	0.889	0.951	0.951	0.954	0.951	0.958
	Length	μ	NA	2.420E+11	5.191	7.951	5.681	8.173	4.575
		σ^2_α	8.032	4.130E+120	13.418	36.809	22.820	44.605	8.978
		$\sigma^2_{\alpha\beta}$	2.433	2.386E+16	2.240	2.300	2.034	2.191	1.724
		σ^2_β	22.983	4.130E+120	16.241	49.944	35.440	73.135	22.205
		σ^2_ϵ	NA	0.464	0.743	0.745	0.742	0.743	0.712
2	Lower	μ	NA	0.912	1.000	1.000	0.999	1.000	0.998
		σ^2_α	0.950	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.946	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_β	0.947	0.999	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.850	0.957	0.957	0.957	0.958	0.942
	Upper	μ	NA	0.917	0.999	1.000	1.000	1.000	0.999
		σ^2_α	0.948	0.938	0.964	0.962	0.968	0.960	0.994
		$\sigma^2_{\alpha\beta}$	0.948	0.871	0.968	0.975	0.976	0.974	0.986
		σ^2_β	0.963	0.981	0.966	0.965	0.970	0.967	0.991
		σ^2_ϵ	NA	0.972	0.947	0.947	0.947	0.945	0.969
	Two	μ	NA	0.881	1.000	1.000	1.000	1.000	1.000
		σ^2_α	0.949	0.973	0.993	0.992	0.996	0.994	0.999
		$\sigma^2_{\alpha\beta}$	0.944	0.904	0.985	0.987	0.990	0.988	0.992
		σ^2_β	0.956	0.990	0.988	0.987	0.990	0.988	0.996
		σ^2_ϵ	NA	0.880	0.956	0.956	0.956	0.957	0.954
	Length	μ	NA	2.626E+10	5.534	8.254	6.204	8.663	4.636
		σ^2_α	10.296	∞	16.404	45.461	30.517	57.685	11.843
		$\sigma^2_{\alpha\beta}$	3.600	1.820E+27	2.923	3.001	2.638	2.876	2.228
		σ^2_β	17.636	∞	17.665	51.585	35.823	68.783	17.113
		σ^2_ϵ	NA	0.734	1.169	1.178	1.168	1.175	1.118
3	Lower	μ	NA	0.923	1.000	1.000	1.000	1.000	1.000
		σ^2_α	0.953	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.948	0.983	1.000	1.000	1.000	1.000	0.999
		σ^2_β	0.955	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.853	0.965	0.965	0.967	0.967	0.952
	Upper	μ	NA	0.925	1.000	1.000	1.000	1.000	0.999
		σ^2_α	0.952	0.921	0.957	0.957	0.965	0.959	0.987
		$\sigma^2_{\alpha\beta}$	0.950	0.954	0.984	0.985	0.986	0.986	0.991
		σ^2_β	0.958	0.924	0.956	0.954	0.966	0.959	0.989
		σ^2_ϵ	NA	0.966	0.942	0.942	0.940	0.940	0.959
	Two	μ	NA	0.905	1.000	1.000	1.000	1.000	1.000
		σ^2_α	0.955	0.964	0.987	0.986	0.991	0.987	0.998
		$\sigma^2_{\alpha\beta}$	0.954	0.976	0.993	0.991	0.993	0.991	0.994
		σ^2_β	0.961	0.962	0.986	0.985	0.988	0.986	0.998
		σ^2_ϵ	NA	0.896	0.956	0.956	0.955	0.956	0.961
	Length	μ	NA	9.666E+09	5.616	8.269	6.114	8.712	4.605
		σ^2_α	12.361	1.330E+133	17.495	49.442	30.913	64.019	14.241
		$\sigma^2_{\alpha\beta}$	5.070	7.320E+33	3.512	3.714	3.187	3.521	2.726
		σ^2_β	12.390	1.520E+171	17.313	49.051	30.944	63.935	13.137
		σ^2_ϵ	NA	0.769	1.244	1.252	1.241	1.248	1.193

TABLE 17
Two-factor crossed with interaction: model design 3 continued.

Par. design	Type	Par.	**HB**	**HLMM**	**BAY**$1_{1.5}$	**BAY**1_3	**BAY**$2_{1.5}$	**BAY**2_3	**FID**
4	Lower	μ	NA	0.939	1.000	1.000	0.999	1.000	0.999
		σ^2_α	0.953	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.950	0.838	0.986	0.992	0.987	0.990	0.970
		σ^2_β	0.945	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.883	0.980	0.981	0.981	0.982	0.972
	Upper	μ	NA	0.942	1.000	1.000	0.999	1.000	0.999
		σ^2_α	0.953	0.905	0.942	0.935	0.949	0.942	0.982
		$\sigma^2_{\alpha\beta}$	0.946	0.975	0.971	0.973	0.976	0.976	0.989
		σ^2_β	0.954	0.901	0.939	0.935	0.947	0.938	0.986
		σ^2_ϵ	NA	0.966	0.933	0.935	0.932	0.935	0.959
	Two	μ	NA	0.922	1.000	1.000	0.999	1.000	1.000
		σ^2_α	0.954	0.952	0.980	0.981	0.984	0.981	0.991
		$\sigma^2_{\alpha\beta}$	0.949	0.939	0.986	0.987	0.986	0.990	0.992
		σ^2_β	0.948	0.943	0.978	0.981	0.985	0.983	0.998
		σ^2_ϵ	NA	0.911	0.957	0.958	0.958	0.959	0.962
	Length	μ	NA	8.610E+10	5.507	8.389	5.970	8.458	4.771
		σ^2_α	13.696	∞	17.237	49.836	32.159	62.968	15.156
		$\sigma^2_{\alpha\beta}$	5.733	5.249E+16	3.996	4.242	3.643	4.050	3.028
		σ^2_β	13.253	∞	17.194	50.279	31.552	63.016	14.598
		σ^2_ϵ	NA	0.515	0.877	0.881	0.879	0.880	0.838
5	Lower	μ	NA	0.918	0.996	1.000	0.999	1.000	0.998
		σ^2_α	0.950	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.955	0.948	1.000	1.000	1.000	1.000	0.997
		σ^2_β	0.945	0.998	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.879	0.974	0.974	0.974	0.973	0.967
	Upper	μ	NA	0.913	0.998	1.000	0.997	0.999	0.994
		σ^2_α	0.952	0.976	0.954	0.951	0.957	0.954	0.986
		$\sigma^2_{\alpha\beta}$	0.949	0.955	0.969	0.969	0.972	0.969	0.982
		σ^2_β	0.951	0.975	0.949	0.944	0.954	0.949	0.984
		σ^2_ϵ	NA	0.964	0.934	0.935	0.935	0.934	0.960
	Two	μ	NA	0.883	0.999	1.000	0.999	1.000	0.999
		σ^2_α	0.958	0.989	0.982	0.984	0.985	0.984	0.997
		$\sigma^2_{\alpha\beta}$	0.951	0.962	0.984	0.986	0.985	0.987	0.993
		σ^2_β	0.949	0.988	0.977	0.977	0.984	0.978	0.997
		σ^2_ϵ	NA	0.901	0.956	0.957	0.955	0.955	0.963
	Length	μ	NA	1.190E+11	10.612	16.282	11.465	16.582	9.459
		σ^2_α	60.565	3.630E+122	63.142	185.177	120.027	233.173	63.488
		$\sigma^2_{\alpha\beta}$	13.792	8.450E+33	11.520	12.412	10.655	11.523	8.776
		σ^2_β	63.786	6.720E+147	63.721	186.809	118.749	244.333	65.051
		σ^2_ϵ	NA	1.678	2.785	2.794	2.792	2.795	2.666

TABLE 18
Two-factor crossed with interaction: model design 4.

Par. design	Type	Par.	**HB**	**HLMM**	**BAY1**$_{1.5}$	**BAY1**$_{3}$	**BAY2**$_{1.5}$	**BAY2**$_{3}$	**FID**
1	Lower	μ	NA	0.911	0.995	0.998	0.993	0.997	0.986
		σ^2_α	0.944	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.957	0.998	1.000	1.000	1.000	1.000	1.000
		σ^2_β	0.947	0.952	1.000	1.000	0.999	1.000	0.981
		σ^2_ϵ	NA	0.813	0.955	0.957	0.960	0.959	0.947
	Upper	μ	NA	0.920	0.995	0.997	0.995	0.998	0.991
		σ^2_α	0.958	0.967	0.961	0.960	0.963	0.960	0.984
		$\sigma^2_{\alpha\beta}$	0.950	0.843	0.970	0.976	0.974	0.973	0.988
		σ^2_β	0.955	0.978	0.951	0.952	0.953	0.953	0.980
		σ^2_ϵ	NA	0.977	0.957	0.959	0.956	0.958	0.969
	Two	μ	NA	0.882	0.998	0.999	0.996	0.999	0.993
		σ^2_α	0.952	0.987	0.984	0.981	0.986	0.982	0.995
		$\sigma^2_{\alpha\beta}$	0.956	0.876	0.987	0.990	0.991	0.989	0.994
		σ^2_β	0.952	0.983	0.979	0.978	0.979	0.979	0.988
		σ^2_ϵ	NA	0.854	0.960	0.964	0.959	0.961	0.962
	Length	μ	NA	1.692E+10	4.397	6.027	4.586	6.029	3.418
		σ^2_α	7.590	3.050E+102	13.589	36.430	21.529	43.407	8.405
		$\sigma^2_{\alpha\beta}$	1.549	2.520E+23	1.510	1.519	1.452	1.499	1.360
		σ^2_β	8.338	1.830E+62	12.350	22.888	14.671	21.584	8.343
		σ^2_ϵ	NA	0.449	0.745	0.748	0.747	0.748	0.710
2	Lower	μ	NA	0.917	0.998	1.000	0.998	1.000	0.997
		σ^2_α	0.945	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.940	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_β	0.944	0.998	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.812	0.951	0.951	0.952	0.952	0.937
	Upper	μ	NA	0.920	0.999	1.000	0.998	1.000	0.997
		σ^2_α	0.956	0.956	0.966	0.961	0.969	0.960	0.990
		$\sigma^2_{\alpha\beta}$	0.946	0.827	0.968	0.971	0.973	0.974	0.988
		σ^2_β	0.956	0.975	0.973	0.971	0.972	0.971	0.992
		σ^2_ϵ	NA	0.979	0.966	0.967	0.965	0.967	0.973
	Two	μ	NA	0.898	1.000	1.000	1.000	1.000	1.000
		σ^2_α	0.949	0.984	0.986	0.987	0.989	0.989	0.996
		$\sigma^2_{\alpha\beta}$	0.944	0.849	0.986	0.987	0.988	0.988	0.994
		σ^2_β	0.949	0.987	0.986	0.987	0.989	0.987	0.997
		σ^2_ϵ	NA	0.856	0.961	0.960	0.961	0.961	0.957
	Length	μ	NA	1.460E+09	4.592	6.318	4.821	6.390	3.474
		σ^2_α	9.146	8.650E+107	15.801	43.365	25.182	53.379	10.443
		$\sigma^2_{\alpha\beta}$	2.264	2.340E+23	1.908	1.921	1.821	1.899	1.694
		σ^2_β	6.316	8.370E+69	11.502	19.280	12.099	17.849	6.363
		σ^2_ϵ	NA	0.691	1.125	1.125	1.122	1.124	1.065
3	Lower	μ	NA	0.943	1.000	1.000	1.000	1.000	0.998
		σ^2_α	0.957	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.947	0.974	0.998	0.999	0.999	0.999	0.998
		σ^2_β	0.947	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.826	0.962	0.962	0.962	0.961	0.949
	Upper	μ	NA	0.938	1.000	1.000	0.999	1.000	0.998
		σ^2_α	0.958	0.934	0.952	0.946	0.959	0.952	0.987
		$\sigma^2_{\alpha\beta}$	0.957	0.914	0.986	0.987	0.990	0.988	0.995
		σ^2_β	0.963	0.890	0.967	0.971	0.973	0.972	0.995
		σ^2_ϵ	NA	0.967	0.958	0.960	0.958	0.960	0.969
	Two	μ	NA	0.926	1.000	1.000	1.000	1.000	1.000
		σ^2_α	0.965	0.964	0.986	0.983	0.984	0.986	0.994
		$\sigma^2_{\alpha\beta}$	0.956	0.941	0.993	0.994	0.996	0.995	0.998
		σ^2_β	0.956	0.935	0.991	0.993	0.993	0.994	0.999
		σ^2_ϵ	NA	0.858	0.955	0.955	0.956	0.956	0.959
	Length	μ	NA	9.981E+16	4.673	6.420	5.040	6.485	3.561
		σ^2_α	11.223	∞	17.384	48.129	32.236	59.101	12.748
		$\sigma^2_{\alpha\beta}$	3.225	9.480E+35	2.345	2.383	2.237	2.342	2.041
		σ^2_β	4.468	5.850E+254	10.259	16.097	10.668	14.688	5.022
		σ^2_ϵ	NA	0.738	1.233	1.239	1.233	1.238	1.154

TABLE 19
Two-factor crossed with interaction: model design 4 continued.

Par. design	Type	Par.	**HB**	**HLMM**	**BAY**$1_{1.5}$	**BAY**1_3	**BAY**$2_{1.5}$	**BAY**2_3	**FID**
4	Lower	μ	NA	0.929	0.997	1.000	0.999	1.000	0.995
		σ^2_α	0.946	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.953	0.821	0.973	0.977	0.973	0.976	0.960
		σ^2_β	0.938	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.851	0.974	0.975	0.978	0.976	0.969
	Upper	μ	NA	0.943	0.999	1.000	0.998	1.000	0.997
		σ^2_α	0.955	0.927	0.936	0.935	0.944	0.938	0.975
		$\sigma^2_{\alpha\beta}$	0.952	0.960	0.981	0.982	0.982	0.984	0.992
		σ^2_β	0.958	0.881	0.959	0.961	0.965	0.965	0.983
		σ^2_ϵ	NA	0.950	0.937	0.938	0.940	0.940	0.954
	Two	μ	NA	0.916	0.999	1.000	0.999	1.000	1.000
		σ^2_α	0.949	0.965	0.976	0.974	0.981	0.973	0.991
		$\sigma^2_{\alpha\beta}$	0.949	0.912	0.984	0.989	0.988	0.989	0.986
		σ^2_β	0.955	0.922	0.986	0.988	0.988	0.988	0.994
		σ^2_ϵ	NA	0.881	0.959	0.959	0.960	0.960	0.965
	Length	μ	NA	1.193E+11	4.513	6.407	4.651	6.352	3.548
		σ^2_α	11.331	1.850E+109	16.617	46.924	28.776	56.834	12.751
		$\sigma^2_{\alpha\beta}$	3.524	2.320E+08	2.542	2.587	2.436	2.535	2.160
		σ^2_β	4.628	7.960E+88	10.145	16.611	10.251	15.261	5.163
		σ^2_ϵ	NA	0.522	0.950	0.952	0.952	0.954	0.876
5	Lower	μ	NA	0.922	0.996	1.000	0.996	1.000	0.990
		σ^2_α	0.950	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.945	0.926	0.995	0.997	0.997	0.998	0.993
		σ^2_β	0.935	0.997	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.849	0.976	0.975	0.976	0.975	0.971
	Upper	μ	NA	0.916	0.997	1.000	0.995	0.999	0.994
		σ^2_α	0.950	0.977	0.941	0.933	0.941	0.935	0.974
		$\sigma^2_{\alpha\beta}$	0.946	0.927	0.980	0.981	0.978	0.980	0.990
		σ^2_β	0.964	0.961	0.965	0.967	0.970	0.967	0.983
		σ^2_ϵ	NA	0.955	0.939	0.940	0.937	0.938	0.954
	Two	μ	NA	0.892	0.997	1.000	0.997	1.000	0.998
		σ^2_α	0.953	0.989	0.968	0.969	0.971	0.968	0.991
		$\sigma^2_{\alpha\beta}$	0.950	0.940	0.992	0.992	0.992	0.991	0.997
		σ^2_β	0.950	0.982	0.983	0.987	0.987	0.987	0.996
		σ^2_ϵ	NA	0.877	0.961	0.961	0.962	0.961	0.962
	Length	μ	NA	1.188E+12	8.960	12.840	9.672	12.754	7.319
		σ^2_α	57.854	2.520E+141	62.950	181.849	122.701	228.830	60.317
		$\sigma^2_{\alpha\beta}$	8.386	1.260E+28	7.221	7.330	6.965	7.226	6.290
		σ^2_β	21.545	2.560E+108	40.073	67.986	43.014	63.561	21.992
		σ^2_ϵ	NA	1.686	2.938	2.945	2.945	2.942	2.742

TABLE 20
Two-factor crossed with interaction: model design 5.

Par. design	Type	Par.	HB	HLMM	BAY$1_{1.5}$	BAY1_3	BAY$2_{1.5}$	BAY2_3	FID
1	Lower	μ	NA	0.914	0.986	0.996	0.989	0.993	0.975
		σ^2_α	0.947	0.998	1.000	1.000	1.000	1.000	0.993
		$\sigma^2_{\alpha\beta}$	0.947	0.951	0.996	0.998	0.998	0.998	0.992
		σ^2_β	0.956	0.914	1.000	1.000	1.000	1.000	0.962
		σ^2_ϵ	NA	0.897	0.959	0.960	0.955	0.957	0.946
	Upper	μ	NA	0.884	0.984	0.994	0.981	0.990	0.970
		σ^2_α	0.957	0.951	0.964	0.967	0.968	0.968	0.977
		$\sigma^2_{\alpha\beta}$	0.946	0.919	0.966	0.965	0.966	0.964	0.968
		σ^2_β	0.939	0.987	0.872	0.865	0.876	0.873	0.948
		σ^2_ϵ	NA	0.965	0.941	0.938	0.940	0.940	0.951
	Two	μ	NA	0.857	0.990	0.999	0.991	0.997	0.981
		σ^2_α	0.956	0.973	0.984	0.984	0.983	0.985	0.989
		$\sigma^2_{\alpha\beta}$	0.949	0.941	0.981	0.983	0.984	0.984	0.981
		σ^2_β	0.947	0.972	0.927	0.922	0.933	0.928	0.971
		σ^2_ϵ	NA	0.917	0.953	0.956	0.956	0.953	0.952
	Length	μ	NA	6.245E+11	4.266	6.127	4.227	6.009	3.535
		σ^2_α	1.415	7.180E+36	3.097	3.254	2.752	2.933	1.370
		$\sigma^2_{\alpha\beta}$	0.724	6.622E+16	0.655	0.658	0.651	0.657	0.579
		σ^2_β	22.661	∞	21.055	59.652	36.647	71.625	22.674
		σ^2_ϵ	NA	0.300	0.372	0.373	0.373	0.373	0.355
2	Lower	μ	NA	0.904	0.994	0.997	0.994	0.998	0.980
		σ^2_α	0.950	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.951	0.997	1.000	1.000	1.000	1.000	1.000
		σ^2_β	0.945	0.994	1.000	1.000	1.000	1.000	0.992
		σ^2_ϵ	NA	0.895	0.960	0.959	0.961	0.959	0.948
	Upper	μ	NA	0.910	0.995	0.998	0.992	0.998	0.983
		σ^2_α	0.954	0.943	0.972	0.975	0.976	0.977	0.984
		$\sigma^2_{\alpha\beta}$	0.962	0.896	0.977	0.980	0.981	0.981	0.979
		σ^2_β	0.952	0.987	0.910	0.905	0.913	0.907	0.956
		σ^2_ϵ	NA	0.961	0.942	0.941	0.942	0.942	0.951
	Two	μ	NA	0.866	0.998	0.999	0.999	0.999	0.992
		σ^2_α	0.955	0.974	0.991	0.990	0.994	0.992	0.997
		$\sigma^2_{\alpha\beta}$	0.962	0.934	0.988	0.992	0.991	0.993	0.992
		σ^2_β	0.949	0.993	0.952	0.945	0.953	0.949	0.983
		σ^2_ϵ	NA	0.919	0.953	0.953	0.952	0.953	0.951
	Length	μ	NA	8.238E+10	4.166	5.914	4.266	5.896	2.919
		σ^2_α	1.611	1.120E+41	3.449	3.574	3.053	3.310	1.554
		$\sigma^2_{\alpha\beta}$	0.964	6.704E+14	0.818	0.821	0.810	0.818	0.720
		σ^2_β	14.162	4.130E+87	20.786	56.094	33.213	67.055	13.854
		σ^2_ϵ	NA	0.487	0.598	0.598	0.599	0.599	0.571
3	Lower	μ	NA	0.912	0.996	1.000	0.996	1.000	0.993
		σ^2_α	0.947	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.944	0.996	0.963	0.967	0.966	0.969	0.942
		σ^2_β	0.954	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.886	0.967	0.966	0.964	0.965	0.954
	Upper	μ	NA	0.908	0.999	1.000	0.996	0.999	0.993
		σ^2_α	0.951	0.971	0.959	0.965	0.962	0.960	0.963
		$\sigma^2_{\alpha\beta}$	0.955	0.978	0.968	0.971	0.969	0.970	0.972
		σ^2_β	0.954	0.912	0.930	0.928	0.932	0.932	0.963
		σ^2_ϵ	NA	0.976	0.935	0.934	0.935	0.935	0.949
	Two	μ	NA	0.864	1.000	1.000	0.998	1.000	0.997
		σ^2_α	0.947	0.989	0.984	0.986	0.985	0.986	0.982
		$\sigma^2_{\alpha\beta}$	0.947	0.985	0.971	0.977	0.977	0.977	0.968
		σ^2_β	0.952	0.951	0.960	0.962	0.963	0.963	0.982
		σ^2_ϵ	NA	0.921	0.956	0.956	0.956	0.956	0.951
	Length	μ	NA	1.641E+11	3.854	5.338	3.938	5.289	2.524
		σ^2_α	2.131	∞	4.614	4.853	3.993	4.757	7.851
		$\sigma^2_{\alpha\beta}$	1.635	2.130E+50	1.277	1.278	1.259	1.276	1.119
		σ^2_β	8.019	∞	18.853	47.673	27.792	51.968	7.851
		σ^2_ϵ	NA	0.744	0.647	0.649	0.647	0.648	0.615

TABLE 21
Two-factor crossed with interaction: model design 5 continued.

Par. design	Type	Par.	**HB**	**HLMM**	**BAY1**$_{1.5}$	**BAY1**$_3$	**BAY2**$_{1.5}$	**BAY2**$_3$	**FID**
4	Lower	μ	NA	0.949	0.997	0.999	0.997	0.999	0.995
		σ^2_α	0.952	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.953	0.787	0.948	0.949	0.945	0.949	0.924
		σ^2_β	0.953	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.925	0.977	0.976	0.977	0.977	0.964
	Upper	μ	NA	0.940	0.998	1.000	0.998	0.999	0.994
		σ^2_α	0.942	0.863	0.939	0.943	0.944	0.945	0.961
		$\sigma^2_{\alpha\beta}$	0.950	0.973	0.962	0.964	0.963	0.964	0.969
		σ^2_β	0.952	0.953	0.928	0.925	0.926	0.927	0.959
		σ^2_ϵ	NA	0.951	0.926	0.925	0.926	0.927	0.933
	Two	μ	NA	0.932	1.000	1.000	0.999	1.000	0.998
		σ^2_α	0.952	0.907	0.974	0.975	0.978	0.976	0.982
		$\sigma^2_{\alpha\beta}$	0.949	0.835	0.957	0.960	0.960	0.960	0.945
		σ^2_β	0.954	0.981	0.960	0.961	0.961	0.963	0.985
		σ^2_ϵ	NA	0.935	0.947	0.945	0.947	0.945	0.946
	Length	μ	NA	5.541E+10	3.707	5.286	3.812	5.078	2.706
		σ^2_α	2.425	∞	5.206	5.777	4.761	5.327	2.420
		$\sigma^2_{\alpha\beta}$	2.079	8.973E-01	1.648	1.660	1.621	1.648	1.443
		σ^2_β	8.760	∞	17.971	46.891	25.867	49.419	8.728
		σ^2_ϵ	NA	0.328	0.422	0.423	0.423	0.423	0.400
5	Lower	μ	NA	0.923	0.989	0.994	0.988	0.996	0.979
		σ^2_α	0.953	0.966	1.000	1.000	1.000	1.000	0.979
		$\sigma^2_{\alpha\beta}$	0.948	0.830	0.967	0.967	0.966	0.967	0.947
		σ^2_β	0.953	0.995	1.000	1.000	1.000	1.000	0.994
		σ^2_ϵ	NA	0.915	0.966	0.965	0.967	0.965	0.954
	Upper	μ	NA	0.925	0.995	0.998	0.989	0.996	0.988
		σ^2_α	0.953	0.959	0.945	0.944	0.945	0.948	0.967
		$\sigma^2_{\alpha\beta}$	0.956	0.957	0.959	0.957	0.957	0.960	0.960
		σ^2_β	0.953	0.988	0.908	0.905	0.919	0.909	0.964
		σ^2_ϵ	NA	0.953	0.936	0.935	0.934	0.933	0.943
	Two	μ	NA	0.897	0.994	0.998	0.989	0.998	0.988
		σ^2_α	0.958	0.973	0.974	0.976	0.975	0.977	0.982
		$\sigma^2_{\alpha\beta}$	0.957	0.886	0.969	0.971	0.969	0.971	0.965
		σ^2_β	0.955	0.995	0.957	0.957	0.956	0.957	0.984
		σ^2_ϵ	NA	0.930	0.947	0.946	0.946	0.945	0.942
	Length	μ	NA	2.610E+11	7.721	11.136	7.711	10.429	5.950
		σ^2_α	11.937	7.990E+59	23.209	27.026	21.063	24.493	11.560
		$\sigma^2_{\alpha\beta}$	4.704	2.275E+00	4.477	4.510	4.431	4.489	3.917
		σ^2_β	49.360	9.090E+126	73.226	198.400	111.236	207.906	48.261
		σ^2_ϵ	NA	1.068	1.362	1.363	1.366	1.365	1.289

TABLE 22
Two-factor crossed with interaction: model design 6.

Par. design	Type	Par.	**HB**	**HLMM**	**BAY1**$_{1.5}$	**BAY1**$_{3}$	**BAY2**$_{1.5}$	**BAY2**$_{3}$	**FID**
1	Lower	μ	NA	0.899	0.990	0.996	0.985	0.996	0.972
		σ^2_α	0.950	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.948	0.991	1.000	1.000	1.000	1.000	1.000
		σ^2_β	0.954	0.956	1.000	1.000	1.000	1.000	0.976
		σ^2_ϵ	NA	0.857	0.962	0.961	0.958	0.960	0.944
	Upper	μ	NA	0.910	0.993	0.999	0.990	0.998	0.982
		σ^2_α	0.952	0.955	0.967	0.967	0.970	0.970	0.984
		$\sigma^2_{\alpha\beta}$	0.958	0.913	0.975	0.977	0.980	0.978	0.979
		σ^2_β	0.961	0.984	0.915	0.907	0.924	0.913	0.968
		σ^2_ϵ	NA	0.961	0.939	0.939	0.937	0.939	0.955
	Two	μ	NA	0.864	0.995	0.999	0.991	0.999	0.988
		σ^2_α	0.953	0.977	0.990	0.991	0.991	0.992	0.998
		$\sigma^2_{\alpha\beta}$	0.954	0.945	0.987	0.989	0.990	0.990	0.990
		σ^2_β	0.953	0.985	0.963	0.958	0.961	0.961	0.981
		σ^2_ϵ	NA	0.888	0.954	0.953	0.953	0.954	0.952
	Length	μ	NA	1.460E+09	4.519	6.588	4.720	6.565	3.624
		σ^2_α	2.484	6.550E+53	6.611	9.660	6.046	8.892	2.451
		$\sigma^2_{\alpha\beta}$	1.031	8.521E+14	0.971	0.976	0.953	0.967	0.813
		σ^2_β	21.258	4.830E+95	18.048	53.761	37.900	71.898	20.943
		σ^2_ϵ	NA	0.404	0.583	0.584	0.585	0.584	0.542
2	Lower	μ	NA	0.913	0.996	0.999	0.995	1.000	0.988
		σ^2_α	0.961	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.947	0.999	1.000	1.000	1.000	1.000	1.000
		σ^2_β	0.955	1.000	1.000	1.000	1.000	1.000	0.998
		σ^2_ϵ	NA	0.850	0.957	0.957	0.958	0.958	0.937
	Upper	μ	NA	0.900	0.999	1.000	0.997	0.999	0.987
		σ^2_α	0.957	0.943	0.978	0.977	0.978	0.979	0.988
		$\sigma^2_{\alpha\beta}$	0.946	0.873	0.974	0.974	0.975	0.975	0.976
		σ^2_β	0.949	0.986	0.927	0.923	0.932	0.927	0.964
		σ^2_ϵ	NA	0.975	0.957	0.957	0.958	0.957	0.965
	Two	μ	NA	0.868	0.999	1.000	0.999	1.000	0.993
		σ^2_α	0.958	0.976	0.993	0.993	0.991	0.993	0.997
		$\sigma^2_{\alpha\beta}$	0.947	0.904	0.986	0.987	0.989	0.990	0.990
		σ^2_β	0.953	0.994	0.965	0.960	0.966	0.962	0.988
		σ^2_ϵ	NA	0.883	0.956	0.954	0.957	0.954	0.955
	Length	μ	NA	3.470E+23	4.576	6.541	4.761	6.470	3.281
		σ^2_α	2.853	2.840E+84	7.745	11.405	7.127	9.808	2.868
		$\sigma^2_{\alpha\beta}$	1.442	6.280E+23	1.245	1.255	1.213	1.246	1.033
		σ^2_β	14.988	1.440E+141	18.898	53.486	32.991	68.278	14.682
		σ^2_ϵ	NA	0.637	0.908	0.908	0.907	0.910	0.843
3	Lower	μ	NA	0.933	0.999	1.000	0.999	1.000	0.994
		σ^2_α	0.952	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.949	0.917	0.992	0.995	0.992	0.994	0.977
		σ^2_β	0.951	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.878	0.971	0.969	0.967	0.971	0.960
	Upper	μ	NA	0.947	1.000	1.000	1.000	1.000	0.998
		σ^2_α	0.947	0.896	0.954	0.955	0.961	0.958	0.976
		$\sigma^2_{\alpha\beta}$	0.960	0.957	0.979	0.981	0.981	0.980	0.983
		σ^2_β	0.953	0.944	0.937	0.937	0.946	0.942	0.974
		σ^2_ϵ	NA	0.960	0.935	0.935	0.935	0.936	0.948
	Two	μ	NA	0.925	1.000	1.000	1.000	1.000	0.999
		σ^2_α	0.947	0.934	0.982	0.985	0.985	0.984	0.990
		$\sigma^2_{\alpha\beta}$	0.950	0.957	0.988	0.989	0.988	0.990	0.987
		σ^2_β	0.953	0.973	0.975	0.974	0.978	0.975	0.985
		σ^2_ϵ	NA	0.910	0.956	0.955	0.953	0.956	0.960
	Length	μ	NA	4.012E+10	4.491	6.098	4.611	6.154	3.038
		σ^2_α	3.801	3.290E+66	9.643	14.039	9.174	12.465	3.830
		$\sigma^2_{\alpha\beta}$	2.305	1.181E+18	1.748	1.803	1.708	1.756	1.469
		σ^2_β	9.324	2.430E+140	17.950	47.543	27.256	57.415	9.328
		σ^2_ϵ	NA	0.688	1.021	1.023	1.021	1.023	0.949

TABLE 23
Two-factor crossed with interaction: model design 6 continued.

Par. design	Type	Par.	**HB**	**HLMM**	**BAY1**$_{1.5}$	**BAY1**$_3$	**BAY2**$_{1.5}$	**BAY2**$_3$	**FID**
4	Lower	μ	NA	0.940	0.999	1.000	0.998	1.000	0.994
		σ^2_α	0.940	1.000	1.000	1.000	1.000	1.000	1.000
		$\sigma^2_{\alpha\beta}$	0.951	0.782	0.952	0.953	0.952	0.955	0.931
		σ^2_β	0.945	1.000	1.000	1.000	1.000	1.000	1.000
		σ^2_ϵ	NA	0.905	0.978	0.977	0.978	0.978	0.971
	Upper	μ	NA	0.933	0.999	1.000	0.999	1.000	0.994
		σ^2_α	0.954	0.888	0.951	0.952	0.957	0.955	0.972
		$\sigma^2_{\alpha\beta}$	0.956	0.978	0.972	0.972	0.974	0.973	0.980
		σ^2_β	0.954	0.938	0.933	0.933	0.939	0.935	0.966
		σ^2_ϵ	NA	0.953	0.919	0.917	0.919	0.916	0.938
	Two	μ	NA	0.921	0.999	1.000	0.999	1.000	0.998
		σ^2_α	0.953	0.933	0.977	0.979	0.980	0.979	0.988
		$\sigma^2_{\alpha\beta}$	0.954	0.859	0.967	0.976	0.971	0.975	0.957
		σ^2_β	0.953	0.968	0.963	0.966	0.973	0.968	0.981
		σ^2_ϵ	NA	0.920	0.946	0.946	0.946	0.947	0.951
	Length	μ	NA	2.153E+10	4.386	6.171	4.545	6.170	3.168
		σ^2_α	4.049	1.380E+99	9.783	15.275	9.154	13.855	4.141
		$\sigma^2_{\alpha\beta}$	2.874	1.092E+00	2.190	2.211	2.107	2.181	1.820
		σ^2_β	9.945	9.650E+110	17.473	47.240	29.046	56.132	9.977
		σ^2_ϵ	NA	0.461	0.733	0.736	0.735	0.735	0.683
5	Lower	μ	NA	0.920	0.996	0.999	0.994	0.999	0.989
		σ^2_α	0.955	0.994	1.000	1.000	1.000	1.000	0.990
		$\sigma^2_{\alpha\beta}$	0.949	0.863	0.985	0.986	0.986	0.988	0.973
		σ^2_β	0.945	0.999	1.000	1.000	1.000	1.000	0.998
		σ^2_ϵ	NA	0.898	0.971	0.972	0.972	0.974	0.960
	Upper	μ	NA	0.940	0.997	1.000	0.997	1.000	0.994
		σ^2_α	0.956	0.964	0.956	0.954	0.957	0.954	0.978
		$\sigma^2_{\alpha\beta}$	0.951	0.948	0.961	0.961	0.961	0.962	0.969
		σ^2_β	0.958	0.980	0.935	0.925	0.936	0.934	0.971
		σ^2_ϵ	NA	0.961	0.928	0.932	0.931	0.932	0.948
	Two	μ	NA	0.906	0.999	1.000	0.998	1.000	0.996
		σ^2_α	0.955	0.983	0.979	0.980	0.981	0.983	0.988
		$\sigma^2_{\alpha\beta}$	0.951	0.927	0.979	0.980	0.978	0.980	0.979
		σ^2_β	0.951	0.990	0.969	0.968	0.971	0.967	0.987
		σ^2_ϵ	NA	0.908	0.951	0.951	0.951	0.950	0.953
	Length	μ	NA	1.760E+10	8.855	12.574	8.936	12.220	6.759
		σ^2_α	20.245	1.530E+75	40.596	68.570	43.793	59.057	19.727
		$\sigma^2_{\alpha\beta}$	6.742	2.120E+01	6.201	6.299	5.994	6.149	5.163
		σ^2_β	52.539	3.120E+209	68.297	189.300	106.988	230.082	51.967
		σ^2_ϵ	NA	1.502	2.311	2.319	2.318	2.315	2.150