STOR 655 – February 25, 2016 Name:______________________

Midterm

All problem parts have equal weight. In budgeting your time expect that some problems will take longer than others.
Remember, answers without proper justification will not receive full credit!

1. Consider the following 99 sorted p-values:
   0.0001 0.0003 0.0005 0.0009 0.0012 0.0025 0.0035 0.0052 0.0068 0.0096
   0.0097 0.0114 0.0116 0.0260 0.0485 0.0805 0.1084 0.1118 0.1247 0.1288
   0.1447 0.1487 0.1541 0.1896 0.1931 0.2187 0.2218 0.2389 0.2592 0.2976
   0.3012 0.3050 0.3054 0.3183 0.3202 0.3481 0.3506 0.3543 0.3738 0.3811
   0.3940 0.3992 0.4185 0.4277 0.4361 0.4436 0.4890 0.4912 0.4954 0.5081
   0.5198 0.5199 0.5232 0.5254 0.5255 0.5292 0.5395 0.5408 0.5629 0.5638
   0.5767 0.5876 0.5960 0.6021 0.6378 0.6438 0.6671 0.6857 0.7085 0.7122
   0.7306 0.7429 0.7495 0.7534 0.7633 0.7653 0.7766 0.7821 0.7828 0.7867
   0.7870 0.8039 0.8084 0.8140 0.8212 0.8229 0.8304 0.8594 0.8698 0.8874
   0.8886 0.9027 0.9066 0.9169 0.9269 0.9330 0.9454 0.9670 0.9970

   How many hypotheses would be rejected at the 1% level using without using any multiple test adjustment. How many would be rejected using Bonferroni adjustment? How many hypotheses would be rejected at the 1% level using the step up and the step down method?

2. Let $X_1, \ldots$ be i.i.d. Poisson(λ).

   (a) Find the asymptotic distribution of $\log(\bar{X}_n + 1)$.

   (b) Find the variance stabilizing distribution for $\bar{X}_n$.

3. (a) Let $X_n \xrightarrow{\mathcal{D}} X$ and $g(s)$ be a continuous function. Prove that $g(X_n) = O_P(1)$.

   (b) Let $X_n \xrightarrow{\mathcal{D}} X$ and $Y_n \xrightarrow{\mathcal{D}} Y$ where X_n and Y_n and X and Y are independent. Prove or disprove that $X_n + Y_n \xrightarrow{\mathcal{D}} X + Y$.

4. Let A be a symmetric matrix. Show that if $\mathbf{X} \sim N_p(\mu, \Sigma)$ and $\mathbf{Y} = \mathbf{X}^T A \mathbf{X}$ then $EY = \text{tr}(A\Sigma) + \mu^T A \mu$.