Parametric Families of Discrete Distributions

Name	pmf $= p_X(x)$	Parameter Space	Mean	Variance	Moment Generating Function $= \mathcal{E}[e^{tX}]$
Discrete uniform	$\frac{1}{n+1} I_{\{0,1,\ldots,n\}}(x)$	$n = 1, 2, \ldots$	$n/2$	$n(n+2)/12$	$\sum_{j=0}^{n} \frac{1}{n+1} e^{jt} = \frac{1-e^{(n+1)t}}{(n+1)(1-e^t)}$
Bernoulli	$p^x(1-p)^{1-x} I_{\{0,1\}}(x)$	$0 \le p \le 1$	p	$p(1-p)$	$(1-p)+p^{t}$
Binomial	$\binom{n}{x} p^x(1-p)^{n-x} I_{\{0,\ldots,n\}}(x)$	$0 \le p \le 1$; $n = 1, 2, \ldots$	np	$np(1-p)$	$[(1-p)+p\,e^t]^n$
Hyper-geometric	$\frac{\binom{K}{x}\binom{M-K}{n-x}}{\binom{M}{n}} I_{\{0,\ldots,n\}}(x)$	$M = 1, 2, \ldots$; $K = 0, \ldots, M$; $n = 1, \ldots, M$	$n\frac{K}{M}$	$n\frac{K}{M}(1-\frac{K}{M})\frac{M-n}{M-1}$	not useful
Poisson	$\frac{e^{-\lambda}\lambda^x}{x!} I_{\{0,1,\ldots\}}(x)$	$\lambda > 0$	λ	λ	$\exp[\lambda(e^t-1)]$
Geometric	$p(1-p)^x I_{\{0,1,\ldots\}}(x)$	$0 < p < 1$	$(1-p)/p$	$(1-p)/p^2$	$\frac{p}{1-(1-p)e^t}$ for $t < -\ell n\,(1-p)$
Geometric	$p(1-p)^{x-1} I_{\{1,2,\ldots\}}(x)$	$0 < p < 1$	$1/p$	$(1-p)/p^2$	$\frac{pe^t}{1-(1-p)e^t}$ for $t < -\ell n\,(1-p)$
Negative Binomial	$\binom{r+x-1}{x} p^r(1-p)^x I_{\{0,1,\ldots\}}(x)$	$0 < p < 1$ and $r > 0$	$r(1-p)/p$	$r(1-p)/p^2$	$[\frac{p}{1-(1-p)e^t}]^r$ for $t < -\ell n\,(1-p)$
Negative Binomial	$\binom{x-1}{r-1} p^r(1-p)^{x-r} I_{\{r,r+1,\ldots\}}(x)$	$0 < p < 1$ and $r > 0$	r/p	$r(1-p)/p^2$	$[\frac{pe^t}{1-(1-p)e^t}]^r$ for $t < -\ell n\,(1-p)$
Beta-binomial	$\binom{n}{x} \frac{B(x+a,n-x+b)}{B(a,b)} I_{\{0,\ldots,n\}}(x)$	$a > 0, b > 0$, $n = 1, 2, \ldots$	$\frac{na}{a+b}$	$\frac{nab(n+a+b)}{(a+b)^2(a+b+1)}$	not useful
Logarithmic	$\frac{(1-p)^x}{(-x\,\ell n p)} I_{\{1,2,\ldots\}}(x)$	$0 < p < 1$	$\frac{(1-p)}{(-p\,\ell n\,p)}$	$\frac{(1-p)(1-p+\ell n\,p)}{-(p\,\ell n\,p)^2}$	$\frac{\ell n\,[1-(1-p)e^t]}{\ell n\,p}$ for $t < -\ell n\,(1-p)$
Discrete Pareto	$\frac{(1/x^{\gamma+1})}{\sum_{j=1}^{\infty}(1/j^{\gamma+1})} I_{\{1,2,\ldots\}}(x)$	$\gamma > 0$	$\frac{\sum_1^{\infty}(1/j)^{\gamma}}{\sum_1^{\infty}(1/j)^{\gamma+1}}$ for $\gamma > 1$		does not exist

Parametric Families of Continuous Distributions

Name	cdf $= F(x)$ or pdf $= f(x)$	Parameter Space	Mean	Variance	Moment Generating Function $= \mathcal{E}[e^{tX}]$
Uniform or rectangular	$f(x) = \frac{1}{\beta} I_{(\alpha,\alpha+\beta)}(x)$	$-\infty < \alpha < \infty$ and $\beta > 0$	$\alpha + \frac{\beta}{2}$	$\beta^2/12$	$\frac{e^{(\alpha+\beta)t} - e^{\alpha t}}{\beta t}$
Normal	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	$-\infty < \mu < \infty$ and $\sigma > 0$	μ	σ^2	$\exp\left[\mu t + \frac{1}{2}\sigma^2 t^2\right]$
Exponential	$f(x) = \frac{1}{\beta} e^{-(x/\beta)} I_{(0,\infty)}(x)$	$\beta > 0$	β	β^2	$\frac{(1/\beta)}{(1/\beta)-t}$ $t < 1/\beta$
Bilateral exponential	$f(x) = \frac{1}{2\beta} e^{-\lvert x-\alpha\rvert/\beta}$	$-\infty < \alpha < \infty$ and $\beta > 0$	α	$2\beta^2$	$e^{t\alpha}/(1 - \beta^2 t^2)$ for $\lvert t\rvert < \frac{1}{\beta}$
Gamma	$f(x) = \frac{1}{\Gamma(r)} \frac{1}{\beta} \left(\frac{x}{\beta}\right)^{r-1} e^{-(x/\beta)} I_{(0,\infty)}(x)$	$r > 0$ and $\beta > 0$	$r\beta$	$r\beta^2$	$\left[\frac{1/\beta}{(1/\beta)-t}\right]^r$ for $t < \frac{1}{\beta}$
Weibull	$f(x) = \frac{\gamma}{\beta} \left(\frac{x-\alpha}{\beta}\right)^{\gamma-1} e^{-\left(\frac{x-\alpha}{\beta}\right)^\gamma} I_{(\alpha,\infty)}(x)$	$-\infty < \alpha < \infty$ and $\beta > 0$ and $\gamma > 0$	$\alpha + \beta\Gamma(1 + \frac{1}{\gamma})$	$\beta^2[\Gamma(1 + \frac{2}{\gamma})$ $-\Gamma^2(1 + \frac{1}{\gamma})]$	not useful; $\mathcal{E}\left[(X-\alpha)^k\right] =$ $\beta^k \Gamma(1 + \frac{k}{\gamma})$
Beta	$f(x) = \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1} I_{(0,1)}(x)$	$a > 0$ and $b > 0$	$\frac{a}{a+b}$	$\frac{ab}{(a+b)^2(a+b+1)}$	not useful; $\mathcal{E}[X^k] =$ $\frac{B(a+k,b)}{B(a,b)}$
Pareto	$f(x) = \frac{\gamma}{(1+x)^{\gamma+1}} I_{(0,\infty)}(x)$	$\gamma > 0$	$1/(\gamma - 1)$ for $\gamma > 1$	$\gamma/[(\gamma-2)(\gamma-1)^2]$ for $\gamma > 2$	does not exist
Cauchy	$f(x) = \frac{1}{\beta\pi} \frac{1}{1+\left(\frac{x-\alpha}{\beta}\right)^2}$	$-\infty < \alpha < \infty$ and $\beta > 0$	does not exist	does not exist	does not exist
Logistic	$F(x) = [1 + e^{-\left(\frac{x-\alpha}{\beta}\right)}]^{-1}$	$-\infty < \alpha < \infty$ and $\beta > 0$	α	$\beta^2\pi^2/3$	$e^{\alpha t}\beta\pi t \csc(\beta\pi t)$
Gumbel or Extreme-value	$F(x) = \exp[-e^{-\left(\frac{x-\alpha}{\beta}\right)}]$	$-\infty < \alpha < \infty$ and $\beta > 0$	$\alpha + \beta\gamma$ where $\gamma \approx .577216$	$\beta^2\pi^2/6$	$e^{\alpha t}\Gamma(1 - \beta t)$ for $t < 1/\beta$
Log normal	$F(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right) I_{(0,\infty)}(x)$	$-\infty < \mu < \infty$ and $\sigma > 0$	$\exp[\mu + \frac{1}{2}\sigma^2]$	$\exp[2\mu + 2\sigma^2]$ $-\exp[2\mu + \sigma^2]$	does not exist $\mathcal{E}[X^k] =$ $\exp[k\mu + \frac{1}{2}k^2\sigma^2]$