STOR 654 – December 10, 2016 Name:________________________

# Final Exam

All problem parts have equal weight. In budgeting your time expect that some problems will take longer than others.
Remember, answers without proper justification will not receive full credit!

1. Let $X_1, \ldots, X_n$ be i.i.d. $N(\theta, \theta^2)$.

   (a) Is this an exponential family?

   (b) Find a minimal sufficient statistics.

   (c) Find the MM of $\theta$. Find the MSE of the MM estimator.

   (d) Find the MLE of $\theta$.

   (e) Find a pivot and use it to find a 95% confidence interval for $\theta$.

   (f) Find a LRT for testing $\mathcal{H}_0 : \theta = \theta_0$ vs. $\mathcal{H}_1 : \theta \neq \theta_0$. What is your rejection rule at level $\alpha = 0.05$? What happens if $\theta_0 = 0$?

2. Let $X_1 \sim \text{Geometric}(p)$ and $X_2 \sim \text{Bernoulli}(p)$, $X_1$ and $X_2$ are independent. Consider the prior $p \sim \text{Beta}(1, 1)$. After deriving the formulas, evaluate all the problem parts for $x_1 = 1$ and $x_2 = 1$.

   (a) Find the posterior distribution

   (b) Find the Bayes estimator (using square loss) of $p$.

   (c) Compute the Bayes factor for testing $\mathcal{H}_0 : p \in (0, 0.5]$ vs. $\mathcal{H}_1 : p \in (0.5, 1)$. What is your decision?

   (d) Propose a UMP test for testing $\mathcal{H}_0 : p = 0.5$ vs. $\mathcal{H}_1 : p = 0.75$. What is your decision at $\alpha = 0.05$?

3. Let $X_1, \ldots, X_n$ be an i.i.d. sample from a distribution with density $\frac{1}{(x-\theta)^2} I_{(1+\theta,\infty)}(x)$.

   (a) Is $EX_k$ finite? What about $EX_{(k)}$?

   (b) Find a p-value for testing $\mathcal{H}_0 : \theta = 1$ vs. $\mathcal{H}_1 : \theta > 1$.