

HOMEWORK SET 9

1. Let $X \sim \text{exp}(1)$ and $Y|X = x \sim \text{uniform}(0, x)$. Find $\rho(X, Y)$.
2. For a parameter $\gamma > 0$ define

$$f_{XY}(x, y) = \frac{\gamma(\gamma + 1)}{(1 + x + y)^{\gamma+2}} \quad x > 0, y > 0.$$

- (a) Find the marginal distributions.
 - (b) What is the distribution of $Y|X = x$.
 - (c) For what values of γ does $\rho(X, Y)$ exist? (Find its value when it exists.)
3. Suppose X and Y have joint pdf given by

$$f_{XY}(x, y) := \begin{cases} 8xy & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $E(X|Y = y)$ and $E(Y|X = x)$.

4. Consider the random variables X and Y where X represents annual return on Treasury bills and Y represents annual return on stocks. Suppose (based on historical data) that the expectations, standard deviations and correlation between these random variables are given by the numbers

$$\mu_X = 5, \sigma_X = 4, \mu_Y = 13.2, \sigma_Y = 17.6, \rho = -.3$$

Suppose you decide to invest 40% of your money in treasury bills and 60% in stocks namely consider the portfolio $R = .4X + .6Y$. What is the **standard deviation** of this random variable R ?

5. Let X_1, X_2, X_3 be independent continuous random variables with common density $f(x)$ and distribution function F . Calculate $P(X_1 < X_2 < X_3)$. (Hint: In the tripple integral make change of variable $u_i = F(x_i)$ for all i .)
6. Let $W \sim \Gamma(a, b)$. Conditional on $W = w$ the $X_1, \dots, X_n|W = w$ are independent $\text{Exp}(w)$. What is the conditional expectation of $E[W|X_1 = x_1, \dots, X_n = x_n]$? (This is related to Bayesian statistics.)

7. The number of hurricanes that will hit a house in the next 10 years has a Poisson distribution with mean 11. Each hurricane results in a loss that is exponentially distributed with rate $\lambda = 1/1250$. Losses are independent and independent of the number of hurricanes. Define the random variable $S =$ total losses due to hurricanes in the next 10 years. Find $E(S)$.