Homework set #8

1. Let $X \sim N(0, \sigma^2)$. Calculate the density of $Y = X^3$.

2. If X_1 and X_2 are independent exponential random variables with respective parameters λ_1 and λ_2, find the distribution of $Z = X_1/X_2$. Also compute $P(X_1 < X_2)$.

3. If X and Y are independent and have independent Exponential(1) find $E(e^{\frac{1}{2}(X+Y)})$.

4. The expected number of typographical errors on a page of a certain magazine is 0.2. What is the probability that an article of 10 pages contains (a) no errors, and (b) 2 or more typographical errors? Explain your reasoning!

5. Jill's bowling scores are approximately normal with mean 170 and standard deviation 20. Jack's scores are approximately normal with mean 160 and standard deviation 15. If Jack and Jill bowl one game, and assuming their scores are independent find the probability that

   (a) Jack's score is higher;

   (b) the total of the scores is higher than 350.

6. The joint pmf of X, Y is given by

$$p(x,y) = \begin{cases} \frac{1}{8} & \text{if } x = 1, 2 \text{ and } y = 1; \\ \frac{1}{4} & \text{if } x = 1 \text{ and } y = 2; \\ \frac{1}{2} & \text{if } x = 2 \text{ and } y = 2; \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Compute the conditional mass function of X given $Y = i$.

   (b) Are X and Y independent?

   (c) Compute $P(XY \leq 3)$, $P(X + Y > 2)$, $P(X/Y > 1)$.

7. Let $X \sim \text{Exp}(\lambda)$ and $[X]$ be the largest integer smaller than or equal to X. Find $P([X] = n, X - [X] \leq x)$. Can you conclude that $[X]$ and $X - [X]$ are independent?

8. The joint density of X and Y is given by

$$f(x,y) = xe^{-x(y+1)}, \quad x > 0,\ y > 0.$$

   (a) Compute the conditional density of X given $Y = y$ and Y given $X = x$.

   (b) Find the density function of $Z = XY$.

9. Define a density

$$f_{XY}(x,y) = Cx^{r-1}y^{s-1}e^{-(x+y+xy)} \quad x > 0,\ y > 0,$$

   where C is a suitable constant and $r, s > 0$.

   (a) Does X have a gamma distribution?

   (b) Does $Y|X = x$ have a gamma distribution?