Homework set #7

1. Let the joint probability mass function of discrete random vector (X, Y) be given by

$$f_{XY}(x, y) = k(x^2 + y^2) \quad x = 1, 2, \; y = 0, 1, 2.$$

   (a) Find the constant k.
   (b) Find the c.d.f $F_{XY}(x, y)$.
   (c) Find $P(X > Y)$, $P(X + Y \leq 2)$, $P(X + Y = 2)$.
   (d) Find the marginal probability mass functions.
   (e) Find $E(XY) - (EX)(EY)$.
   (f) What is the distribution of $X + Y$, $X - Y$ and XY.

2. Let the joint density of continuous random vector (X, Y) be given by

$$f_{XY}(x, y) = k(x^2 + y^2) \quad x \in (1, 2), \; y \in (0, 2).$$

   (a) Find the constant k.
   (b) Find the c.d.f $F_{XY}(x, y)$.
   (c) Find $P(X > Y)$, $P(X + Y \leq 2)$, $P(X + Y = 2)$.
   (d) Find the marginal densities.
   (e) Find $\rho(XY) = \frac{E(XY) - (EX)(EY)}{\sqrt{\operatorname{var}(X) \operatorname{var}(Y)}}$.

3. Let X, Y be discrete. Prove that X and Y are independent if and only if $p_{X,Y}(s, u) = p_X(s) p_Y(u)$ for all s, u.