

HOMEWORK SET #6

1. The annual rainfall in a certain area is normally distributed with mean 40 and standard deviation 4. What is the probability that starting with next year, it will take over 10 years before a year with annual rainfall over 50 occurs? What assumptions are you making?
2. Let X be a normal random variable with mean 5. If $P(X > 9) = .2$, what is $\text{var } X$?
3. Let $X \sim N(10, 36)$. Compute
 - (a) $P(X > 5)$
 - (b) $P(4 < X < 16)$
 - (c) $P(X < 20)$
4.
 - (a) A fire station is to be located along a road of length $0 < A < \infty$. Fires occur at points chosen uniformly on $(0, A)$, where should be the fire station located to minimize expected distance to the fire? That is, if $X \sim U(0, A)$ choose a so that $E|X - a|$ is minimized.
 - (b) Now suppose that the length is infinite and fire occur at random points that are distributed exponentially with rate λ . Where should be the fire station located in this case? That is, if $Y \sim \text{Exp}(\lambda)$ choose a so that $E|Y - a|$ is minimized.
5. Let X be exponential random variable with mean $1/\lambda$. Find EX^k .
6. Show $\Gamma(1/2) = \sqrt{\pi}$.
7. Let X be a random variable that takes values in $[0, c]$, i.e., $P(0 \leq X \leq c) = 1$. Show $\text{var } X \leq \frac{c^2}{4}$. (Hint: $EX^2 \leq cEX$.)