Homework set #5

1. The Maxwell distribution has

$$f_X(x;\beta) = \begin{cases} \frac{4}{\beta\sqrt{\pi}} \left(\frac{x}{\beta}\right)^2 \exp\left[-\left(\frac{x}{\beta}\right)^2\right] & \text{for } x > 0, \\ 0 & \text{otherwise,} \end{cases}$$

   with $\beta > 0$. Find its mean and variance.

2. Consider a function

$$f(x) = \begin{cases} C(2x - x^3) & \text{if } 0 < x < 2.5 \\ 0 & \text{otherwise.} \end{cases}$$

   Could $f(x)$ be a density? If yes determine C.

3. The probability density function of X, the lifetime of a certain electronic device (in hours) is given by

$$f(x) = \begin{cases} \frac{10}{x^2} & \text{if } x > 10 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find $P(X > 20)$.
   (b) What is the distribution function of X?
   (c) What is the EX?
   (d) What is the probability that out of 6 such devices at least 3 will function for at least 15 hours? What assumptions are you making?

4. You arrive at a bus stop at 10 o'clock knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30.

   (a) What is the probability you have to wait longer than 10 minutes?
   (b) It is is 10:15 and and the bus has not yet arrived. What is the probability you will have to wait at least another 10 minutes?

5. Jan's daughter Klára is going to a birthday party in a neighboring town. Suppose that if she is s minutes early for the party, then she incurs an emotional cost $2s$ (time spent twiddling thumbs looking uncomfortably at hosts, the cupcakes etc). If she is s minutes late, then she incurs a cost $4s$ (missing out the funnest part of the party, if you are too late cliques already form and you stand around looking uncomfortably at all the groups already having tons of fun). Suppose also that the travel time X measured in minutes from where she is to the location of the party is a continuous random variable having probability density function

$$f_X(x) = \begin{cases} \frac{1}{1800}(60 - x), & \text{for } 0 < x < 60, \\ 0 & \text{otherwise} \end{cases}$$

Determine the time at which Klára should depart if he wants to **minimize** her expected cost. You can use the expression we derived in class and go from there.

6. Assume that Y has density $f(y)$. Show that

$$EY = \int_0^\infty P(Y > y)\, dy - \int_0^\infty P(Y < -y)\, dy.$$