Homework set #4

1. If $EX = 1$ and $\operatorname{Var} X = 5$ find

   (a) $E[(2+X)^2]$,

   (b) $\operatorname{Var}(4+3X)$.

2. A total of 4 buses is carrying 148 students arrives at a football stadium. The busses carry 40, 33, 25, and 50 students respectively. One student and one bus driver is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student and Y denote the the number of students that were on the bus driven by the randomly selected driver.

   (a) Which of EX and EY do you thin is larger? Why?

   (b) Compute EX and EY.

   (c) Find $\operatorname{Var} X$ and $\operatorname{Var} Y$.

3. Let N denote the number of heads occurring in n tosses of a biased coin. ($P(H) = p = 1 - q$.)

   (a) What is the probability mass function of N? What is $E(N)$ and $\operatorname{Var}(N)$?

   (b) Find the probability p_n that N is even? (Hint: **Prove** and utilize the following identity $\sum_i \binom{n}{2i} x^{2i} y^{n-2i} = \frac{(x+y)^n + (y-x)^n}{2}$.)

   (c) Is there a random variable Y such that

   $$P(Y = n) = \begin{cases} \frac{Cp_n}{2^n} & \text{for } n = 1, 2, 3, \dots; \\ 0 & \text{otherwise?} \end{cases}$$

   If yes, find C.

4. Let $X \sim \mathrm{B}(n, p)$. Show that

   $$E\left[\frac{1}{X+1}\right] = \frac{1 - (1-p)^{n+1}}{(n+1)p}.$$

5. Show:

$$\lim_{r\to\infty,\ p\to 1,\ r(1-p)=\lambda} \binom{r+x-1}{x} p^r (1-p)^x = \frac{e^{-\lambda}\lambda^x}{x!}$$

for $x = 0, 1, 2, 3, \ldots$. (This is a Poisson approximation to negative binomial.)

6. Ler $X \sim$ Poisson(λ). Show $EX^n = \lambda E[(X+1)^{n-1}]$. Use this to compute EX^3.

7. Prove

$$\sum_{i=0}^{n} \frac{\lambda^i e^{-\lambda}}{i!} = \frac{1}{n!} \int_{\lambda}^{\infty} e^{-x} x^n \, dx.$$

8. Suppose X is a Poisson random variable with mean λ. If $P(X = 1 | X \leq 1) = .9$ what is the value of λ?