Homework set #3

1. Suppose there are $n+1$ urn labeled $0, 1, 2, \ldots, n$, and the ith urn contains i green and $n-i$ white balls. An urn is selected at random, and then we sample the balls from the selected urn one-by-one with replacement. If the first m balls are green, what is the probability that the next ($m+1$st) ball will be green as well? Find the limit as $n \to \infty$.

2. In the famous "Monty Hall game" there are 3 doors. Monty Hall offers you the opportunity to win what is behind one of the three doors. Typically there was a really nice prize (ie. a car) behind one of the doors and a not-so-nice prize (ie. a goat) behind the other two. After selecting a door, Monty would then proceed to open one of the doors you didn't select. It is important to note here that Monty would NOT open the door that concealed the car. If he had a choice he would open the door on the left with probability p and the door on the right with probability $1-p$. At this point, he would then ask you if you wanted to switch to the other door before revealing what you had won. Is it to your advantage to switch? (Hint: The answer might depend on p.)

3. Suppose A, B and C are **independent** events. Assume $P(A) = 2/3$, $P(B) = 3/4$ and $P(C) = 1/5$. Find the probability of the following two events:

   (a) $A \cap B^c \cap C^c$.

   (b) Exactly two of the three events occur.

4. Suppose you toss a fair coin three times. Which of the following pairs of events are independent? Give math justification for your answer (namely if you claim that A and B **are independent** then show that $P(A \cap B) = P(A)P(B)$; if you claim they are **not** independent, show that $P(A \cap B) \neq P(A)P(B)$).

   (a) A = "heads on first toss"; B= "an odd number of heads".

   (b) A = "no tails in the first two tosses"; B= "no heads in the second and third toss".

5. A small college town is supplied electricity by a company called **Fluke energy**. This company has 10 different power generators supplying electricity to the town. Power plant i fails with probability p_i independently for different power plants (so one power plant failing does not affect the probability of another power plant failing).

   Assume that any one power generator produces enough power to supply the entire town. What is the probability that the town has a blackout? Here p_i are some fixed numbers between 0 and 1. Your answer will be a formula in terms of the $p_i's$.

6. We roll a die n times. Let $A_{i,j}$ be the event that ith and jth roll produce the same number. Are the events $\{A_{i,j}, 1 \leq i < j \leq n\}$ independent?

7. The following is called "Galton's paradox". I toss three coins. Two must be the same. The probability that the third one lands on the same side as the other two is 1/2. Therefore $P(\text{“all alike”}) = 1/2$. Do you agree with this reasoning?