Homework 2

1. A conference room contains m men and w women. These people seat at random in $m+w$ seats arranged in a row. Find the probability that all the women will be adjacent.

2. You ask your neighbor to water a sickly plant while you are on vacation. Without water, it will die with probability .8; with water, it will die with probability .15. You are 90 percent certain that your neighbor will remember to water the plant.

   (a) What is the probability that the plant will be alive when you return?

   (b) If the plant is dead upon your return, what is the probability that your neighbor forgot to water it?

3. Urn I and Urn II each contains 3 red and 3 white balls. First we transfer one ball from Urn I to Urn II. Then we transfer one ball from Urn II to Urn I. Finally we sample one ball from Urn I and it is red. What is the probability the both transferred balls were also red?

4. Is it possible that there are A, B, C such that $P(A) < P(B)$ and at the same time $P(A|C) > P(B|C)$ and $P(A|C^{\complement}) > P(B|C^{\complement})$? If it is possible show an example if it is not possible prove it is not possible.

5. An urn contains b blue and r red balls. They are removed one by one at random and not replaced.

   (a) Show that the probability that the first time red ball drawn is the $(k+1)$th ball equals $\frac{\binom{r+b-k-1}{r-1}}{\binom{r+b}{b}}$.

   (b) Find the probability that the last ball drawn is red.

6. **Prenatal Screening:** Please see the book *Risk Savvy* for more details on the data below. Much of this comes from the UK.

   In many countries, pregnant women above age of 35 advised by doctors to attend screening for Down syndrome. Risk of bearing a child with Down syndrome does not *suddenly* increase at age 35 but increases smoothly with age. Risk estimated to be:

- For 30 year olds: 1/1000.
- For 35 year olds: 3/1000.
- For 40 year olds: $10/1000 \equiv 1\%$.

As one might imagine, this is very stressful for women. There are **two tests**. a first trimester blood test for screening. If this is positive then second *invasive* Chorionic villus sampling (CVS) test (sampling of the placental tissue).

(a) First trimester test: sonogram to detect excess skin on necks and tests for serum markers. If test is positive need to decide if they want to do second test.

(b) Risk of miscarriage in 2nd test: estimated to be around 1/200.

(c) **Main point:** If first test is positive should women decide to take second test?

Data Relevant for the problem: Now suppose you are a medical professional and you are advising a **40 year old** pregnant woman. Data Relevant for this population (as per the Risk Savvy book):

- About 1% chance of babies with Down syndrome.
- If baby has Down syndrome: 90% chance first trimester test positive.
- If baby not affected: still 5% chance first trimester test is positive.

Suppose 40 year old pregnant woman tests positive, what is the chance baby actually has Down syndrome?

7. An experiment consists of drawing 10 cards from an ordinary 52-card deck. Recall this deck consists cards of 13 different denominations: A, 2-10, J, Q, K and four suits: ($\clubsuit, \diamondsuit, \heartsuit, \spadesuit$).

   (a) If the drawing is made *with* replacement, find the probability that no two cards are of the same denomination.

   (b) If the drawing is made *without* replacement, find the probability that at least 9 cards will have the same suit.

8. Find a formula for the event $P(A|B^c)$ only in terms of $P(A)$, $P(B)$ and $P(A \cap B)$.