Homework 1

1. Five separate awards (best scholarship, best leadership qualities, and so on) are to be presented to selected students from a class of 20. How many different outcomes are possible if

   (a) a student can receive any number of awards?

   (b) each student can receive at most 1 award?

2. People have come up with various nice "laws" when studying coincidences, see for example the beautiful book *Improbability principle* by David Hand. One of the laws is called the *Law of Inevitability* namely in an experiment: **something must happen**. At first sight this sounds trite but let us see it in action. In 1990, the Virginia state lottery was being conducted as follows. You have to choose 6 **distinct** numbers from the numbers 1 to 44; the **order does not** matter so one lottery ticket could be:

   23, 06, 43, 11, 37, 09

   Suppose each ticket is worth \$1. Suppose you wanted to buy all possible tickets. How much would you have to spend?

   History time: In Feb 1992 since no one had won in the previous weeks, the jackpot had ballooned up to \$ 27 million. A group called "International Lotto fund" compromising around 2500 small investors (mainly from Australia but also US, Europe, NZ) raised the money and bought all possible combinations and won!

3. 6 women and 6 men are shipwrecked on a tropical island. How many ways can they

   (a) Form 6 male-female couples. Note here a couple of the form $\{Jim, Ann\}$ is the same as $\{Ann, Jim\}$.

   (b) Form 6 male-female couples and get married if we keep track of the order in which the weddings occur.

4. Consider the set $S = \{T, HT, HHT, HHHT, \ldots\}$ be the set of possible outcomes when you keep tossing a coin till the first time you see a tail (so e.g. HHT represents the first toss was Head, the second was head and the third was tail so you stopped the experiment). For $k \geq 1$, let E_k denote the event that you need **at least** k tosses to get a tail. Write down all possible outcomes in the event:

$$E_2 \cap E_5^c$$

5. You ask a friend to choose an integer N between -5 and 5 (here -5 and 5 are included). Let $A = \{N \leq 3\}$, $B = \{3 \leq N \leq 7\}$ and $C = \{N \text{ is even and } > 0\}$. List the outcomes that belong to the following events:

   (a) $A \cap B \cap C$

   (b) $A \cup (B \cap C^c)$

   (c) $(A \cup B) \cap C^c$

   (d) $(A \cap B) \cap ((A \cup C)^c)$

6. One event operation that we did not discuss in class is the "set-minus" operation. For two event E and F, the event $E \setminus F$ is the event consisting of all outcomes that are in E but not F. Answer **yes** or **no** to the following: For 3 events A, B, C is the following identity true:

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$

7. DELETED

8. Five fair *dice* are tossed once.

   (a) What is the probability of a "full house"?

   (b) What is the probability of "exactly two pairs"?

9. A fair die is tossed n times. What is the probability of the event at east one "face" never appears? (Intuitively this probability should go to 0 as n goes to infinity. Check yours.)