Homework set #5

1. Assume that $X$ is $N(0, 1)$, $B$ satisfies $P(B = 1) = P(B = -1) = 1/2$, and $B$ is independent of $X$. Define $Y = B |X|$. 

   (a) Find the distribution of $Y$.
   (b) Find $\text{Cov}(X, Y)$.
   (c) Are $X$ and $Y$ independent? Explain.

2. Let $Y \sim \chi^2(p, \lambda)$.

   (a) Show that if $Y \sim \chi^2(p, \lambda)$ then $EY = p + \lambda$, $\text{Var}(Y) = 2(p + 2\lambda)$.
   (b) Find the moment generating function of $Y$.

3. Show that if $Y_1 \sim \chi^2(p_1, \lambda_1)$ and $Y_2 \sim \chi^2(p_2, \lambda_2)$ are independent, then $Y_1 + Y_2 \sim \chi^2(p_1 + p_2, \lambda_1 + \lambda_2)$.