1. Assume $(X_1, Y_1), \ldots, (X_n, Y_n)$ are i.i.d. bivariate normal \( \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \rho & \rho \\ \rho & 1 \end{pmatrix} \right) \), \( 0 < \rho < 1 \).

   (a) Consider \( R_n = \frac{1}{n} \sum_{i=1}^{n} X_i Y_i \) and \( V_n = \frac{1}{n} \sum_{i=1}^{n} X_i^2 \). Are they consistent estimators of \( \rho \)?

   (b) Compute the asymptotic relative efficiency \( ARE(R_n, V_n) \). Do you prefer one of the statistic over the other? Why?

   (c) Find the asymptotic distribution of \( \frac{R_n^2}{V_n} \). Is it a consistent estimator of \( \rho \)?

   (d) Is any of the three estimators of \( \rho \) you computed above asymptotically efficient? If no, suggest an asymptotically efficient estimator; you do not have to obtain a closed form expression. (Hint: \( \frac{1}{\rho} + \frac{1}{1-\rho} = \frac{1}{\rho - \rho^2} \).)
More room for your solutions.
2. Let $X \sim \text{Binomial}(n, p)$ and consider the prior $p \sim \text{Uniform}(0, 1)$

(a) Find the MLE and the Bayes estimator (using square loss) of $p$ and evaluate them for $n = 50, x = 50$.

(b) Find the 90% HPD credible interval for $p$ if you observe $n = 50, x = 50$.

(c) Find the Bayes factor for testing $H_0 : p \in (0, 0.95]$ vs. $H_1 : p \in (0.95, 1)$. Evaluate the Bayes factor for $n = 50, x = 50$.

(d) Modify the prior to test $H_0 : p = 0.95$ vs. $H_1 : p \neq 0.95$ and find the Bayes factor. Evaluate the Bayes factor for $n = 50, x = 50$.

(e) Find the likelihood ratio test for testing $H_0 : p = 0.95$ vs. $H_1 : p \neq 0.95$. Evaluate the value of the test statistic and find an approximate p-value for $n = 50, x = 50$. 

More room for your solutions.
3. Let $X_1, X_2, \ldots$ be i.i.d. random variables so that $EX_i^6 < \infty$. Denote 
\[ \mu = EX_i \text{ and } \mu_j = E(X_i - \mu)^j. \]

(a) Find a U statistic for estimating $\mu_3$.

(b) Find the Hájek projection of your U-statistic.

(c) Find the asymptotic distribution of your U statistic.

(Hint: Use the the result of part b. If you did not find a solution to part b, you may use 
\[ n^{-1} \sum_{i=1}^{n} [(X_i - \mu)^3 - 3\mu_2(X_i - \mu) \mu_1] - \mu_3. \])