1. Is it possible for $X, Y, Z$ to have the same distribution and satisfy $X = U(Y + Z)$, where $U \sim U(0, 1)$, and $Y, Z$ are independent of $U$ and each other?

2. Let $X \sim \text{Bin}(n, p)$. Find the MGF of $X$. Show that for $\epsilon > p$

$$P(X/n \geq \epsilon) \leq \exp \left\{ -n(1 - \epsilon) \log \left( \frac{1 - \epsilon}{1 - p} \right) - n\epsilon \log \left( \frac{\epsilon}{p} \right) \right\}$$

3. Prove that for all $x > 0$

$$\left( \frac{1}{x} - \frac{1}{x^3} \right) \phi(x) < 1 - \Phi(x) < \frac{1}{x} \phi(x).$$

4. Let $h(u)$ be the function appearing in the version of Bennett’s inequality given in class. Show, as claimed, that $h(u) \geq u^2(2 + 2u/3)^{-1}$.

5. Let $Z_1, \ldots, Z_n$ be i.i.d. $N(0, \sigma^2)$ and write $U_i = \max(\min(Z_i, 1), -1)$. In what follows $t > 0$.

(a) Prove $P(\sum_{i=1}^n Z_i > nt) \leq \frac{\sigma \exp(-nt^2/2)}{\sqrt{2\pi n t}}$.
(b) Prove $P(\sum_{i=1}^n U_i > nt) \leq \exp(-nt^2/2)$.
(c) Find $EU_i$ and prove $\text{Var} U_i \leq \sigma^2$.
(d) Prove $P(\sum_{i=1}^n U_i > nt) \leq \exp(-\frac{nt^2}{2\sigma^2 + 4t/3})$. 